ESTIMATION OF A DISEQUILIBRIUM
AGGREGATE LABOR MARKET

by

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I. Introduction*

One of the most important questions in contemporary economics is whether or not the real wage clears the labor market. Its answer has bearing on issues as diverse as the nature of unemployment, the impact of fiscal and monetary policies, and the incidence of income taxes. Unfortunately, consensus as to the correct answer seems to be lacking. While much of modern macroeconomic theory assumes that the real wage fails to equate the supply and demand of labor [3], a good deal of work is based on the assumption of equilibrium in the labor market [22]. The purpose of the present paper is to carry out an econometric test for which view of the labor market is more appropriate. The tentative conclusion is that the hypothesis of a labor market in continuous equilibrium must be rejected.

The word 'tentative' in the last sentence must be emphasized. As will be discussed below, the model we build is very aggregative and much too crude to be used as a basis for policy. We believe, however, that it is an improvement upon earlier econometric studies of the labor market, and provides a first step in making operational the theoretical literature on disequilibrium macro models.

In Section II we describe briefly some earlier work on modelling the aggregate supply and demand for labor. It is shown that prior studies either assume equilibrium in the labor market, or deal with disequilibrium inadequately.

In Section III we specify the disequilibrium model. Section IV contains a discussion of the econometric problems involved in estimating it, and an interpretation of the results. An equilibrium version of the model is also estimated, and the results analyzed. A concluding section has a summary and an agenda for future research.

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II. Antecedents

In the present section we discuss several attempts to estimate the parameters of an aggregative model of the labor market. 1 Attention is focused on how the possibility of disequilibrium in the labor market is handled in each study. In this paper, 'disequilibrium' describes a situation in which price fails to equate supply and demand. This usage is in marked contrast to that of (e.g.) Nadiri and S. Rosen [20]. They characterize a model as being in disequilibrium if the economic actors fail to reach an optimum in a given period, even though supply and demand are always equal.

An early (1958) attempt to estimate the supply and demand of labor is that of Mosbaek [19]. His model consists of a labor force participation equation for the entire economy, and a demand equation for labor in the bituminous coal industry. Labor force participation is a function of the average wage, consumer price index, time, and a dummy for World War II. Demand is a function of the wage, the wholesale price index, time, and the World War II dummy. The model is estimated by ordinary least squares using annual time series data. Interestingly, the justification for ignoring simultaneous equations problems is the fact that "... the labor market is not in equilibrium" ([19], p. 140).

The use of ordinary least squares in such a context is inappropriate (see, e.g., [26]), and it seems curious to estimate the supply curve for the entire economy jointly with the demand curve for the coal industry. The Mosbaek model is of interest, then, not because of its statistical methodology, but because it is one of the few attempts to estimate a labor market model based on a

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1 Lucas and Rapping [18] provide a concise and valuable summary of attempts to estimate labor supply functions prior to the late 1960's. Summaries of some of the best examples of work on labor supply since then can be found in [13]. The literature on the demand for labor is discussed by Nadiri and S. Rosen [20] and Hamermesh [14].
standard supply and demand framework.

In the Black-Kelejian (B-K) model [4], an aggregate constant elasticity of substitution (CES) production function is assumed, and the demand for labor is derived from the marginal productivity condition for profit maximization.\(^2\) The supply side of the model is represented by two labor force participation rate equations, one for primary and one for secondary workers. The real wage does not appear as an argument in either of these equations.\(^3\)

The B-K model also includes a wage adjustment equation: "... in each period a disequilibrium position is assumed to exist in the labor market, thus generating a wage adjustment (p. 716) ... [P]ercentage changes in money wages ... are related to the rate of unemployment ... and to percentage changes in both product prices and the marginal productivity of labor ..." (p. 717). The simultaneous system is estimated by two-stage least squares with quarterly time series data.

A major problem arises with respect to B-K's treatment of disequilibrium. In equilibrium models, observed quantities are at the intersections of supply and demand. In a disequilibrium model, however, a given observation is on one curve or another, not both. In general, it cannot be known \textit{a priori} which schedule an observation lies upon. This inherent problem in a disequilibrium model is not considered by B-K.

The last model we shall consider is Lucas and Rapping's (L-R) [18], which is perhaps the most famous and important attempt to estimate the supply and

\(^2\)They also include an equation to explain the division of total manhours between number of workers and hours per worker.

\(^3\)The variables included are a time trend, total employment divided by total labor force, ratio of manufacturing employment to total private employment, a dummy to correct for changes in census definitions, and the ratio of labor compensation less taxes plus transfers to household wealth. (The last of these appears only in the equation for primary earners.)
demand for labor. Although we cannot do justice to the rich theoretical detail of the L-R model, we describe the main components of its structure. The aggregate supply of labor depends upon current and anticipated wages and prices, the interest rate, and the market value of household assets. The 'demand' side of the model is derived from the marginal productivity condition for a CES production function. There is no disequilibrium in the model: "... The current wage is assumed to equate quantity demanded and quantity supplied each period." (p. 272). Nevertheless, L-R do allow for unemployment, and posit that it is due to job search: "Because information is limited and costly to acquire and because action on the basis of acquired information sometimes requires large resource investments in moving and retraining, the suppliers of labor will adjust slowly" (p. 273). Using this theory of unemployment and making certain simplifying assumptions, L-R complete their model with an equation which relates the unemployment rate to current and lagged wages and prices and to the lagged unemployment rate. The model is estimated with annual data from 1929 to 1965, a period that includes the Great Depression.

There is one serious problem with the L-R model. In both the supply and demand equations, great care is taken to account for the fact that behavior adjusts slowly. It is emphasized that in any given period, suppliers and/or demanders may not be at their long-run optimal positions. In the presence of all these lags, it appears inconsistent to take as a maintained hypothesis that the labor market itself is always in equilibrium. With this assumption, L-R avoid the econometric problems associated with disequilibrium, but at the cost of imposing a possibly unrealistic constraint upon their model. The model

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L-R properly stress that since output, which is endogenous, appears as an explanatory variable, a marginal productivity condition for labor is quite different from a demand function for labor.
developed in the next section is similar in some respects to that of L-R, but it allows for the possibility that the labor market may fail to equilibrate.

III. The Model

In this section we construct a model of the labor market which is simple, yet based upon microeconomic foundations. As noted earlier, there are problems in interpreting the parameters of aggregative models, but past experience has shown that estimates of macroeconomic functions are nevertheless both interesting and useful.

The model consists of four equations: one each for the marginal productivity of labor, the supply of labor, the observed quantity of labor, and the real wage adjustment. Each of these is discussed in turn.

A. Marginal Productivity of Labor

The theory of factor demand suggests that the marginal productivity of labor in period $t$ can be written

$$\text{MPL}_t = f(Q_t, L_t, t) \quad (1)$$

where $\text{MPL}_t$ is the marginal product of labor in period $t$, $Q_t$ is output, $L_t$ is manhours of labor, and $t$ is a time trend representing the state of technical progress in period $t$. A necessary condition for profit maximization is that the marginal product equal the real wage ($w_t$):

$$w_t = f(Q_t, L_t, t) \quad (2)$$

Assuming that (2) can be solved for $L_t$,

$$L_t^D = \delta(w_t, Q_t, t) \quad (3)$$

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5 Some of these problems are discussed in [26].
The superscript \( D \) indicates that this is the quantity demanded conditional on output, but it should be noted that (3) is not a true demand equation because of the appearance of \( Q_t \) on the right hand side.

For purposes of estimation, a specific functional form must be given to (3). It is simplest to use a log linear formulation (except for \( t \)), which is also a first-order Taylor Series approximation:

\[
\ln L_t^D = \alpha_0 + \alpha_1 \ln w_t + \alpha_2 \ln Q_t + \alpha_3 t
\]

(4)

Ideally, one would want to study a multi-market model in which output was an endogenous variable. This is a task which must be accomplished eventually, but it is beyond the scope of the current study. As noted above, the assumption that output is exogenous is common to most earlier studies in this and related areas.

B. Supply of Labor

The labor supply function is based upon the theory of leisure-income choice. For an individual with a net wage \( w_{nt} \) and net unearned income \( A_{nt} \), the indirect utility function can be written

\[
v = v(w_{nt}, A_{nt})
\]

(5)

---

If the underlying production function is assumed to be CES, the coefficients in (4) can be used to solve the following set of interdependent equations for the CES parameters: \( \alpha_1 = -\sigma \), \( \alpha_2 = (\sigma h + 1 - \sigma)/h \), and \( \alpha_3 = -\lambda(1 - \sigma)/h \), where \( \sigma \) is the elasticity of substitution, \( h \) measures returns to scale, and \( \lambda \) is the rate of Hicks-neutral technological change. Although this is an interesting interpretation, the usefulness of (4) does not rest upon the CES specification.

For example, it is a common assumption in studies of investment demand. See [16].

The classic exposition of this theory is found in Robbins [25].
where \( V(\cdot) \) is the indirect utility function. Employing Roy's Identity, (5) can be used to find the utility maximizing number of hours of work:

\[
L_t^S = h(w_t, A_t). \tag{6}
\]

This model is atemporal in the sense that the leisure-income choice in period \( t \) depends only upon the wage and unearned income in that period. This is considerably simpler than the model of Lucas-Rapping, and saves us from having to make arbitrary assumptions with respect to expectations mechanisms. Since our observations are annual, this does not seem too severe a restriction.

Given that the population changes over time, (6) must be augmented by a scale variable which captures changes in the size of the potential labor force. Calling the potential number of manhours \( P_t \) and using a log-linear functional specification, (6) becomes

\[
\ln L_t^S = \beta_0 + \beta_1 \ln w_t + \beta_2 \ln A_t + \beta_3 \ln P_t. \tag{7}
\]

A number of points need to be made with respect to this specification.

Although \( A_t \) is assumed to be exogenous, in reality it depends upon past hours of work. Therefore, its estimated coefficient is likely to be biased. Similarly, \( P_t \), which is driven mainly by population growth, would be endogenous in a more detailed model. There is also a problem in the interpretation of the supply elasticity with respect to the wage. Since there are different 'types' of labor in the economy, \( \beta_1 \) must be viewed as a complicated weighted average of different groups' responses to changes in the net wage.

C. Observed Quantity of Labor

In an equilibrium model, the observed quantity of labor is given by the intersection of the supply and demand curves. In a disequilibrium model, this is not the case. We will assume, as does much of the recent work in disequilibrium
theory [3], that the quantity observed is the minimum of quantity supplied and quantity demanded at the current wage:

\[ \ln L_t = \min(\ln L^S_t, \ln L^D_t). \]  

(8)

Clearly, (8) is not the complete story. At least one problem is that it does not explain how rationing takes place. Moreover, if aggregation is over sub-markets some of which are characterized by excess demand and some by excess supply, the observed quantity of labor might be some combination of \( L^S_t \) and \( L^D_t \). Despite its simplicity, however, we regard the 'min condition' as a reasonable first approximation for characterization of a disequilibrium market.

D. Real Wage Adjustment

Standard Walrasian analysis suggests that if the wage fails to clear the labor market during a given period, the forces of supply and demand will tend to move it toward equilibrium, ceteris paribus. Unfortunately, at this time, economic theory tells us little about exactly why the wage is 'sticky,' or the determinants of the speed at which it moves toward equilibrium. Search costs and uncertainty are probably key parts of the answer. The fact that wage contracts take time to negotiate is also important. To begin, we assume that the change in the logarithm of gross wage is a constant proportion of the discrepancy between the logarithm of quantities demanded and supplied:

\[ \ln w_t - \ln w_{t-1} = \gamma (\ln L^D_t - \ln L^S_t), \]  

(9)

\footnote{Despite their widespread use in both empirical and theoretical work, 'min conditions' such as (8) have been subject to some criticism. For example, Phelps and Winter [23] build a model of temporary monopoly in competitive markets which has the implication that min conditions are obsolete for modelling disequilibrium phenomena. The relevance of their model is not clear, particularly for analysis of the labor market. It is not evident, for instance, that individual firms can be viewed as monopsonists, even for short periods of time. For some econometric uses of min conditions see [10] and [12].}
where $\gamma_1 > 0$.

It is possible that noncompetitive elements in the economy can induce movements in the real wage independent of changes in supply and demand. For example, the presence of unions is often thought to influence real wages, although the precise direction of this effect is logically indeterminate.\footnote{If unions manage to increase the real wage of their members, those who lose jobs in the unionized sector may enter the nonunion sector, driving down the real wage there. The net effect depends upon the magnitudes of the relevant behavioral and technological elasticities. See [18], p. 262.} It has been argued by Hines [15] that wage changes are influenced by the rate of change in the degree of unionization of the labor force. Although we depart from the precise formulation of his relationship, we accept it in principle and augment (9) with a variable $U_t$, the percent of the labor force which is unionized in period $t$:

$$
\ln w_t - \ln w_{t-1} = \gamma_1 (\ln L^D_t - \ln L^S_t) + \gamma_2 U_t .
$$

(10)

In the results reported below, we experiment with several other variables which might control for the impact of noncompetitive factors on real wage adjustment.

E. Summary

Previous models have either assumed equilibrium in the labor market or failed to deal adequately with the implications of disequilibrium. In this section we have developed a simple model of the labor market, appealing to microeconomic principles to guide the treatment of disequilibrium. The supply and marginal productivity equations are based upon standard choice theoretic considerations. The 'min condition' and wage adjustment equations follow from the textbook version of Walrasian supply and demand analysis. We now turn to the econometric problems involved in estimating the model, and comparing it to the equilibrium counterpart.
IV. Estimation and Results

In this section estimates of the model specified above are presented. We begin by discussing the data, and then outline the estimation procedure. The parameter estimates and their implications are analyzed at some length. A number of variants of the basic model are tested, including one in which equilibrium is taken as a maintained hypothesis. Perhaps the most interesting conclusion to emerge is that the disequilibrium model appears to be more compatible with the data than its equilibrium analogue.

A. Data

For purposes of reference we restate the model:

\[ \ln L_t^D = \alpha_0 + \alpha_1 \ln w_t + \alpha_2 \ln Q_t + \alpha_3 t + \epsilon_{1t} \]  \hspace{1cm} (11)

\[ \ln L_t^S = \beta_0 + \beta_1 \ln w_{nt} + \beta_2 \ln A_{nt} + \beta_3 \ln P_t + \epsilon_{2t} \]  \hspace{1cm} (12)

\[ \ln L_t = \min(\ln L_t^S, \ln L_t^D) \]  \hspace{1cm} (13)

\[ \ln w_t - \ln w_{t-1} = \gamma_1 (\ln L_t^D - \ln L_t^S) + \gamma_2 U_t + \epsilon_{3t} \]  \hspace{1cm} (14)

Equations (11), (12) and (14) differ from their counterparts above only by the addition of the error terms \( \epsilon_{1t} \), \( \epsilon_{2t} \) and \( \epsilon_{3t} \), whose joint distribution is specified below.

The data are annual observations on the U.S. economy for the years 1930 through 1973. \( L_t \) is total private hours worked per year expressed in billions. It is the product of private domestic hours per person and number of persons engaged in private production.\(^{11}\) \( w_t \) is total wages and salaries in the private sector expressed in 1958 dollars, divided by the number of private

\(^{11}\)The numbers used are from an updated and revised version of the series found in Table 4 of [7]. We are grateful to L. Christensen and D. Jorgenson for making these unpublished figures available to us.
hours worked. $Q_t$ is gross national product expressed in billions of 1958 dollars.\textsuperscript{12}

The net wage $w_{nt}$ is the product of the gross wage $w_t$ and a factor $(1-\theta_t)$, where $\theta_t$ is the ratio of personal taxes to personal income in period $t$.\textsuperscript{13}

This adjustment for taxes is similar to that used by Abbott and Ashenfelter [1]. Undoubtedly, more sophisticated corrections for taxes could be made, but this simple one seemed adequate for our purposes. $A_{nt}$ is the sum of rent, dividends, interest, and profits\textsuperscript{14} in 1958 dollars divided by the number of workers, adjusted for taxes by multiplying by $(1-\theta_t)$. $A_{nt}$ does not include retained earnings or the imputed income from durables; in the absence of a consensus as to how such complications should be treated, it seemed best to use the simplest definition possible.

$P_t$ is the potential number of hours of work available in year $t$ expressed in billions. It is calculated by multiplying the number of civilians between the ages of 16 and 64 by the average number of hours worked per year.\textsuperscript{15} The implicit assumption behind this formulation is that in any given year, those absent from the labor force can potentially contribute an annual number of hours equal to the average of those in the labor force. An alternative specification is simply to set $P_t$ equal to the number of civilian individuals between 16 and 64; this possibility is also considered below. A more refined version of $P_t$ might include corrections for demographic changes in the labor force.

\textsuperscript{12}The consumer price index is used to convert to 1958 dollars. Figures for total wages and salaries and GNP are from [9] and [29].

\textsuperscript{13}Data for personal taxes and personal income are from [9] and [29].

\textsuperscript{14}Rent, dividends, profits, and interest are from [9] and [29].

\textsuperscript{15}The population series is from [9].
force. This would be an interesting problem for future research.

Finally, \( U_t \) is the percentage of the work force unionized in year \( t \).

**B. Stochastic Specification and Estimation**

We assume that the error terms \( \varepsilon_{it} (i=1,2,3) \) are distributed normally with mean zero and variance-covariance matrix

\[
\Sigma = \begin{bmatrix}
\sigma_1^2 & 0 & 0 \\
0 & \sigma_2^2 & 0 \\
0 & 0 & \sigma_3^2
\end{bmatrix}
\]

Given that the error terms are normally and independently distributed, Equs. (11), (12) and (14) define the joint density function of the endogenous variables \( \ln L_t^D, \ln L_t^S \) and \( \ln w_t \). Denoting this density function by \( g(\ln L_t^D, \ln L_t^S, \ln w_t) \), it is straightforward to show that the joint density of the observable random variables \( \ln L_t, \ln w_t \) is

\[
h(\ln L_t, \ln w_t) = \int_{\ln L_t}^{\infty} g(\ln L_t, \ln L_t^S, \ln w_t) d\ln L_t^S + \int_{\ln L_t}^{\infty} g(\ln L_t^D, \ln L_t, \ln w_t) d\ln L_t^D \]  

(15)

The likelihood function is obtained from (15) as

\[
L = \prod_{t=1}^{T} h(\ln L_t, \ln w_t) \]  

(16)

For a detailed derivation of (15) and (16) the reader is referred to [24].

Maximum likelihood is the estimation technique.\(^1\) The asymptotic standard errors of the estimates are computed by taking the square roots of the diagonal elements of the negative inverse Hessian matrix of the loglikelihood function.

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\(^{16}\) Source: [5], Table 1.58.

\(^{17}\) The numerical optimizations were performed using the Davidson-Fletcher-Powell and the quadratic hill-climbing algorithms [11].
C. Results

The parameter estimates are shown in column 1 of Table 1. Examining first
the outcomes for the demand equation, we note that $\alpha_1$ and $\alpha_2$, the
elasticities of quantity demanded of labor with respect to the real wage and
output, are close to unity in absolute value ( $\alpha_2$ is slightly in excess of two
standard deviations away from one). These values are within the range of other
estimates of the labor marginal productivity condition.\textsuperscript{18} The coefficient of
the time trend has the expected negative sign, but it is statistically
insignificant from zero.\textsuperscript{19}

Turning now to the supply of labor, we first note that the elasticity with
respect to the net wage ($\beta_1$) is small in absolute value and insignificantly
different from zero. This result is common in virtually all time series and
many cross section studies,\textsuperscript{20} and suggests that the income and substitution
effects of real wage changes are approximately offsetting. An apparently counter-
intuitive result is the sign of $\beta_2$, which suggests a positive elasticity with
respect to unearned income. This phenomenon has been noted in a number of
previous studies. For example, when Lucas and Rapping [18] include real non-
human wealth per household in their model, the coefficient is insignificant,
and in one of their equations it is positive. [8] report a similar result from
an analysis of cross section data. This may be due to the facts that a) high
asset income is associated with high hours of work in the past, and b) there is

\textsuperscript{18} A summary of these results is presented by Hamermesh [14], pp. 512, 516.

\textsuperscript{19} If the marginal productivity relation is assumed to be derived from a CES
production function, the implied parameters are $\sigma = .984 (.105)$, $h = .114 (.826)$,
and $\lambda = .027 (.047)$. See footnote 6. Observe that if $\alpha_2$ were 1.0, the point
estimate of $h$ would also be 1.0. Due to the extreme nonlinearity of the
relation between $h$ and $\alpha_2$ around the point $h = 1$, even small deviations
away from $\alpha_2 = 1$ (such as our value of 1.095) lead to drastic changes in the
value of the returns to scale parameter. This phenomenon is reflected in the
large standard error associated with $h$.

\textsuperscript{20} See, for example, [18] or [13]. Of course, various subgroups in the population
may exhibit labor force behavior quite responsive to changes in the real net wage.
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<td>( a_0 )</td>
<td>-1.330 ( \pm .199 )</td>
<td>-1.413 ( \pm .357 )</td>
<td>-1.460 ( \pm .261 )</td>
<td>-2.442 ( \pm .985 )</td>
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<td>( a_1 )</td>
<td>-0.984 ( \pm .105 )</td>
<td>-0.982 ( \pm .097 )</td>
<td>-0.974 ( \pm .094 )</td>
<td>-1.480 ( \pm .452 )</td>
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<td>( a_2 )</td>
<td>1.095 ( \pm .038 )</td>
<td>1.111 ( \pm .069 )</td>
<td>1.122 ( \pm .051 )</td>
<td>1.241 ( \pm .196 )</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>-0.003 ( \pm .003 )</td>
<td>-0.003 ( \pm .003 )</td>
<td>-0.004 ( \pm .003 )</td>
<td>0.012 ( \pm .019 )</td>
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<td>( \beta_0 )</td>
<td>0.209 ( \pm .496 )</td>
<td>3.422 ( \pm .313 )</td>
<td>0.059 ( \pm 1.196 )</td>
<td>3.616 ( \pm .904 )</td>
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<td>( \beta_1 )</td>
<td>0.008 ( \pm .040 )</td>
<td>-0.019 ( \pm .057 )</td>
<td>-0.0001 ( \pm .051 )</td>
<td>0.015 ( \pm .075 )</td>
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<td>( \beta_2 )</td>
<td>0.490 ( \pm .046 )</td>
<td>0.610 ( \pm .062 )</td>
<td>0.492 ( \pm .048 )</td>
<td>0.526 ( \pm .075 )</td>
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<td>( \beta_3 )</td>
<td>0.871 ( \pm .091 )</td>
<td>0.347 ( \pm .070 )</td>
<td>0.899 ( \pm .219 )</td>
<td>0.216 ( \pm .168 )</td>
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<td>( \gamma_1 )</td>
<td>0.182 ( \pm .058 )</td>
<td>0.228 ( \pm .073 )</td>
<td>0.125 ( \pm .056 )</td>
<td>-</td>
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<td>( \gamma_2 )</td>
<td>0.002 ( \pm .0003 )</td>
<td>0.001 ( \pm .0003 )</td>
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<td>197.22</td>
<td>199.91</td>
<td>178.30</td>
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*Variables are defined in the text. Numbers in parentheses are standard errors.*
serial correlation in hours of work. In an experiment not reported here, we
substituted a net wealth variable for $A_{nt}^*$, and this result was essentially
unchanged.

$\beta_3$, the elasticity of number of hours worked with respect to the potential
number of hours of work, is about .87. The estimated standard error indicates
that it does not differ significantly from one. This is in accord with prior
expectations -- if the population were doubled, one would expect, ceteris paribus,
to see the desired quantity of labor supplied also double. This suggests that
failure to correct for demographic changes in the labor force may not be too
serious a problem, although this matter needs further investigation.

Finally, consider the parameters of the wage-adjustment equation. The
coefficient on excess demand, $\gamma_1$, is positive and differs significantly from
zero, precisely as theory suggests. (We discuss below the implied speed of
adjustment toward equilibrium.) As noted above, the sign of $\gamma_2$, the coefficient
on the unionization variable, is logically indeterminate, although in a somewhat
similar formulation Hines [15] had a positive coefficient. It turns out to be
positive and statistically significant, and implies that an increase of ten
percentage points in the labor force that is unionized would lead to a two
percent annual increase in real wages, ceteris paribus.

In summary, the parameter estimates of the disequilibrium model generally
accord with a priori expectations. The supply and demand parameters are quanti-
tatively in line with the results of earlier studies. In particular, they do
not differ greatly from Lucas and Rapping's long run elasticities, despite the
fact that our underlying equations contain no lagged variables.\footnote{In terms of our notation, L-R find $a_1 = -1.09, a_2 = 1.00$. Compare these
with the results from Table 1: $a_1 = -.984, a_2 = 1.095.$} Since there
are no wage-adjustment equations such as (14), in the earlier literature,
comparisons are impossible, but the results certainly do not seem unreasonable.

D. Further Discussion

A number of variations on the basic model were investigated in order to ascertain its sensitivity to changes in specification. One experiment was to substitute the number of civilian individuals of age 19 through 64 for potential number of hours in the supply equation. The results, which are shown in column 2 are somewhat less satisfactory than those for the basic model. The coefficient on the population variable falls to .347, and the elasticity of labor supply with respect to unearned income becomes even more positive. The usual likelihood ratio test cannot be used to discriminate between the models of columns 1 and 2 because they express alternative hypotheses that are not nested with respect to one another. On a somewhat heuristic level it is comforting, however, that the less reasonable parameter estimates are accompanied by a lower likelihood.

Another variable examined was the unionization rate in the wage-adjustment equation. It can be argued that \( U_t \) may be proxying for a whole set of characteristics which influence the real wage independently of supply and demand, e.g., industrial concentration. Suppose that these noncompetitive forces tend to increase the real wage by a certain percentage each year independent of supply and demand. The implications of such a view can be investigated simply by substituting a constant (1.0) for \( U_t \) and re-estimating the system. The results are shown in column 3. The coefficient \( \gamma_2 \) implies that even with equality of supply and demand, there is a tendency for the real wage to increase by about three percent annually.\(^{22}\) The likelihood ratio is somewhat lower than

\(^{22}\) Several other variations on the basic model were investigated. In one, a binary variable was added to the supply equation to shift the intercept during the World War II years. In another, nonlabor income was omitted from the supply equation. These results, which are available from the authors upon request, did not have a major impact on the basic qualitative conclusions.
in column 1 but so slightly that even a heuristic comparison of likelihoods is difficult. 23

Another important aspect of the wage adjustment equation is the implication that excess demand and the unionization variable induce changes in the real wage. An alternative hypothesis, based on the observation that wage bargains are made in nominal terms, is that the right hand variables affect the money wage. Denoting the appropriate price index by \( c_t \) and the money wage by \( w_t^m \), we have \( w_t = \frac{w_t^m}{c_t} \) and (14) becomes

\[
\ln w_t^m - \ln w_t^{m-1} = \gamma_1 (\ln L_t^D - \ln L_t^S) + \gamma_2 U_t + \epsilon_{3t}
\]
or

\[
\ln w_t - \ln w_{t-1} = - (\ln c_t - \ln c_{t-1}) + \gamma_1 (\ln L_t^D - \ln L_t^S) + \gamma_2 U_t + \epsilon_{3t} \quad (14')
\]

We may now introduce a new coefficient \( \gamma_3 \) into (14') which becomes

\[
\ln w_t - \ln w_{t-1} = \gamma_1 (\ln L_t^D - \ln L_t^S) + \gamma_2 U_t + \gamma_3 (\ln c_t - \ln c_{t-1}) + \epsilon_{3t} \quad (14'')
\]

If excess demand affects the money wage, \( \gamma_3 = -1 \); if it affects the real wage, \( \gamma_3 = 0 \). The models given in columns 1 and 2 were reestimated using (14'') as the price adjustment equation. Comparing the results with columns 1 and 2 gives -2 \( \ln \) (likelihood ratio) of 3.2 and 3.1 respectively; using \( \chi^2(1) \) these are not sufficiently large at the .05 level to reject \( \gamma_1 = 0 \). (The same result holds if we examine \( \gamma_3 \) directly for departures from zero). At the same time we emphatically reject the hypothesis that \( \gamma_3 = -1 \). Hence the adjustment process in real wages is confirmed.

A related question is whether workers correctly perceive the effect of taxes

23 An additional variant is estimation with the variable \( U_t \) replaced by a time trend. In this case the loglikelihood is slightly higher than in column 1 but the coefficient estimates are barely changed.
when making their work decision, i.e. whether it is the gross or net wage that enters the supply of labor function. As (12) stands, it is assumed that the net wage is the appropriate variable. However, several investigators have suggested that individuals fail to take taxes into account. (See [2] or [6]).

In order to examine this issue, \( w_t \) and \( (1-\theta_t) \) were entered as separate variables in (12) and the system re-estimated. The coefficients of \( w_t \) and \( (1-\theta_t) \) differed negligibly and -2 times the loglikelihood ratio for testing the null hypothesis that these coefficients were equal yielded a test statistic of 2.56. Thus, the hypothesis that the coefficients of these variable were equal could not be rejected, confirming that it is the net wage that matters.

Perhaps the most interesting question associated with the model is whether or not it is 'better' than its equilibrium counterpart. This is not a straightforward problem, because the hypothesis of equilibrium is not strictly speaking a nested hypothesis.\(^{24}\) Nevertheless, an approximate test can be made by estimating the equilibrium version and applying the likelihood ratio test. It is also of considerable interest to ascertain the extent to which parameter estimates change when equilibrium is assumed.

The equilibrium system consists of the marginal productivity condition for labor (11), the supply equation (12), and the market clearing relation

\[
\ln L_t = \ln L_t^D = \ln L_t^S.
\]  

(17)

Maximum likelihood estimation of such a system is a familiar problem (see, e.g., [26]). The estimates corresponding to the disequilibrium model in column 1 are in column 4. The comparison of the results with those in column 1 points toward disequilibrium as the favored hypothesis: (a) The parameter estimates in column

\(^{24}\) It is nested only asymptotically. See [24] for a discussion.
4 are less reasonable (\(a_1\) is numerically too large, \(a_3\) has the wrong sign and \(\beta_3\) is much too small); (b) -2 ln (likelihood ratio) is 48.7, rejecting equilibrium strongly. Finally, (c) it has been suggested [24] that \(1/\gamma_1\) be examined for significant departures from zero: if such were found, the equilibrium model would tend to be confirmed. In the present case \(1/\gamma_1\) divided by an estimate of its standard error is 1.75 for column 1 and 1.40 for column 2; not large enough to give support to the equilibrium hypothesis.

Two aspects of the model remain to be explored. The first is the speed with which the system moves toward equilibrium after it is shocked. Assume that all the exogenous variables in the system are held constant and that error terms are zero. Then it can be shown that the steady state value of the logarithm of the gross wage is

\[
\ln w_t = \psi k_1^t + \frac{k_2 t}{c} + \frac{k_3}{c} - \frac{k_2}{c^2}
\]  

(18)

where

\[
k_1 = 1/(1-(a_1-\beta_1)\gamma_1)
\]  

(19)

\[
k_2 = \gamma_1 a_3/(1-(a_1-\beta_1)\gamma_1)
\]  

(20)

\[
k_3 = (\gamma_1(a_2-\beta_2) + \gamma_2)/(1-(a_1-\beta_1)\gamma_1)
\]  

(21)

\[
c = 1 - k_1
\]  

(22)

and \(\psi\) is a constant which depends upon initial conditions. Substituting the estimates from column 1 into (19) yields \(k_1 = .847, k_2 = -.005, k_3 = .095\). Equ. (18) thus is \(\ln w_t = \psi(.847)^t - .003t + .619\). Disregarding the autonomous part of the solution, one observes from the transient part that an initial discrepancy between \(\ln w_t\) and its equilibrium path will be reduced by 50
percent in about 4 periods. This conclusion, however, is rather sensitive to which version of the model is employed: similar calculations for column 2 indicate a speed of adjustment about twice as rapid. In any event, however, the value of \( k_1 \) appears to be sufficiently high to suggest that adjustment is sluggish and that the estimated disequilibrium model is not, in fact, mimicking the behavior of an equilibrium model.

The second issue concerns the implications for estimates of 'true' unemployment rates. As Lucas and Rapping ([18], p. 274) emphasize, the question used in unemployment surveys does not ask individuals whether or not they are seeking employment at the current market wage. It is therefore possible that individuals are counted as unemployed even though they are unwilling to work at the going wage. In short, measured unemployment rates probably do not yield a measure of involuntary unemployment as the concept is commonly used in economic theory.

However, one can develop such a measure by using the information reported in Table 1. For each year substitute the values of the right hand side variables into (11) and (12), thus generating predictions of \( L^D_t \) and \( L^S_t \), respectively. The expression \( \hat{R}_t = (\hat{L}_t^S - \hat{L}_t^D)/\hat{L}_t^S \) (where \( \hat{\cdot} \) denotes a predicted magnitude) provides an estimate of the rate of involuntary unemployment in year \( t \). It is further possible, by employing a Taylor series expansion of \( \hat{R}_t \) about the parameter estimates, to obtain approximate expressions for the expected value of \( \hat{R}_t \), \( E(\hat{R}_t) \), and for its standard deviation, \( \sigma(\hat{R}_t) \).

The point estimates for \( R_t \) were disappointing, since positive estimated excess demand appears in all years up to 1946. The remaining years show results that are not counterintuitive: 1946, 1947 and 1949 exhibit small excess supplies; excess demand reigns in 1948 and 1950-53; finally, the period 1954-73 is characterized by excess supply. The point estimates may not be sufficiently accurate, however, to distinguish periods characterized by common
measures as periods of excess demand from those of excess supply. For this reason one may examine quasi-confidence bounds given by \( E(R_t - 3 \hat{R}_t), E(R_t + 3 \hat{R}_t) \). For any period there are three possible outcomes: (a) the lower limit is positive, in which case the period is classified as one of excess demand; (b) the upper limit is negative, in which case the period is one of excess supply; (c) the lower limit is negative and the upper limit positive in which case the classification is ambiguous. According to this criterion 1931, 1933-35, 1947-50 and 1953-55 are ambiguous, 1930, 1932, 1936-45, 1951-52 are excess demand periods, and the remaining years, 1946 and 1956-73 excess supply periods. It is disappointing that the years of the Great Depression do not show up more unambiguously on the excess supply side, although the excess demand during World War II and the Korean War as well as the excess supply in the last decade do make sense. It is clear that further study is needed to investigate the ability of disequilibrium models to predict the values of unobservable endogenous variables.

V. Concluding Remarks

We have specified and estimated a simple aggregative disequilibrium model of the labor market. The supply and demand equations are based upon choice theoretic considerations, and the wage adjustment mechanism follows from standard Walrasian analysis. The supply and demand elasticities are generally in accord with those of earlier studies, but the evidence suggests that the equilibrium framework of these studies was inappropriate. Movement of the system toward equilibrium is quite sluggish.

As suggested throughout this paper, the labor market is only one part of a general equilibrium system. The other markets must be included in the model in order to obtain more reliable estimates. The theoretical and statistical
problems involved are formidable, but not insurmountable. It is hoped that the current paper will provide an impetus for the required research.

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25 Some of these problems are described in [24].
Bibliography


