PARETO INFERIOR TRADE AND OPTIMAL TRADE POLICY

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One of the basic tenets of liberal economics is that everyone could be made better off by reducing restrictions on trade. Even though some groups in the population may be made initially worse off, those who gain as a result of trade liberalization can more than afford to compensate those who suffer.

We show that in a competitive but risky economy, free trade may be Pareto inferior to no trade, and under fairly general conditions a restriction in trade from the free-trade position will constitute a Pareto improvement. One result consistent with traditional beliefs is that there exists some form of liberalization from the no-trade situation which constitutes a Pareto improvement.

It has already been noted in the literature that tariffs and quotas are not equivalent in the presence of uncertainty (Fishelson and Flatters [1975], Dasgupta and Stiglitz [1977]). However, the conditions under which one or the other is preferable are far from well understood. We extend the analysis of the comparison of quotas and tariffs by establishing, in our model, that tariffs are preferable to quotas near the no-trade equilibrium, but quotas are preferable to tariffs near the free trade equilibrium. If one views the current situation to be one "closer" to free trade than to no-trade, then our model provides some justification for the seemingly prevalent agitation for the use of quotas rather than tariffs.
The basic idea behind our model is simple. There are two countries (regions) both of which grow a risky agricultural crop and a safe crop. The output in the two regions is negatively correlated. (The results can easily be extended to cases where the correlation is zero or even positive, so long as the correlation is not perfect.)

In the absence of trade, price rises whenever output falls. If demand functions have unitary price elasticity the price variations provide perfect income insurance for the farmer. But with the opening of trade, because of the negative correlation between output in the two regions price variations no longer offset output variations in each country, and so the riskiness of growing the crop faced by each farmer is increased. This induces farmers to shift production away from the risky crop, raising its average price. However, in the special model we examine, where consumers have unit price elasticity and thus constant expenditure on both crops, the mean income of the farmers remains constant with the opening of trade. Since its riskiness increases, it follows that farmers' welfare necessarily decreases, as shown in Figure 1.

Whereas before trade was opened, consumers bore all the risk, with free trade they bear none, and, other things being equal, this would make them better off. However, the change in the pattern of supply, and the rise in the average price of the risky crop, makes them worse off. Near autarky, the risk benefit dominates this allocation effect, as shown in Figure 1, but near free trade the opposite is the case. If the change in supplies and prices is sufficiently large (which it will be if producers are sufficiently risk averse) and if the consumer risk benefits are sufficiently small (i.e. if consumers are not very
Figure 1  Welfare Consequences of Opening Trade
risk averse) then consumers will be made worse off by opening trade. Since producers are necessarily worse off (in this model) it follows that free trade is Pareto inferior to autarky.

The reconciliation of our results with the standard theorems of Welfare Economics in which free trade is Pareto efficient is straightforward; the conventional argument requires not only that markets be competitive (as we assume), but also complete. In our model there must be a complete set of insurance markets enabling farmers to purchase both price and output insurance. For a variety of reasons such as moral hazard and adverse selection, the set of markets is not complete. Essentially what happens in our example is that while autarky provides farmers with income insurance, when a new market is opened (in this case international trade markets in the risky commodity) the commodity market no longer provides implicit insurance, and it is as though insurance markets were closed. If there is a complete set of markets then each market provides one, and only one, marketing service, but if there is not a complete set then some markets may be providing several services, allocating both goods and risk. Institutional change may change the number of services provided by a particular market, and in our example such changes can make everyone worse off. Welfare analysis which assumes that each market serves only one function can be seriously misleading in these cases.

The model we shall analyse has been deliberately made simple to make the main points as clearly as possible. In particular, we choose very special functional forms--unit price elasticity--for consumer demand which allow us to employ the concept of mean preserving changes in risk, and which thus allow a very intuitive explanation of our results. It will become clear from our
model that any other specification would greatly complicate the analysis, since changes in trade will in general lead to changes in both mean income and its risk.¹ In fact, only three assumptions are critical for our results. First, there must be some agents who, on average are net sellers of the risky good (in our model, the farmers) and others who are net buyers (consumers). Since these individuals must engage in trade, their welfare is affected by the price distribution. Second, neither producers nor consumers can buy insurance for the risks (the variability of output of the risky crop and of its price) which they face. Then changes in the level of trade change the price distribution and the risks that individuals face. If they could perfectly insure, then there would be no change in risk, and hence no adverse consequences. However, we would argue that it is more realistic to assume that individuals cannot perfectly insure themselves, and that therefore risk, and changes in risk, do matter. Thirdly, the output in the different countries cannot be perfectly correlated, for then (under our assumptions that the countries are otherwise identical) opening up trade would leave the price distribution unaffected. Thus, although the assumption we employ here, that outputs in the two countries are perfectly negatively correlated is admittedly extreme, all that is required is the absence of perfect correlation. In part III we defend the robustness of our results at greater length.

¹. Elsewhere (Newbery-Stiglitz 1981b) we have demonstrated the inefficiency (except under very stringent conditions) of competitive (i.e. free trade) equilibria for a more general model. The results that there exists some intervention to the free trade equilibria derived in the present paper are robust, and would continue to apply in more general models.
It is these three assumptions which in fact distinguish this paper from earlier studies of trade and uncertainty. These other studies have (for the most part) employed the concept of a representative consumer-producer, assumed the price distribution was given exogenously, rather than determined endogenously, assumed outputs are perfectly correlated across countries, and/or focused on the case where there are perfect risk markets, so farmers can diversify out of the risks they face. Perhaps the closest in spirit to the present study is the excellent work of Helpman and Razin (1978) who are concerned with general equilibrium effects; but they assume a perfect equities market (a weaker assumption than a perfect risk market), perfect correlation, and they make use of the concept of a representative consumer-producer.

2. For an extensive survey of the recent literature on trade and uncertainty, the reader is referred to Pomeroy (1979).

Among the studies focusing on the small country case with the price distribution given exogenously are those of Batra and Russell and Ruffin (1974). The Batra and Russell and Ruffin papers also assumed producers had no output variability; the relationship between the variability of output and prices plays a crucial role in our analysis.

In addition to the two papers cited earlier, the literature comparing tariffs and quotas in the presence of uncertainty includes Pelcovits (1976), Young (1979) and Young and Anderson (1980). Like the studies cited earlier, they employ a partial equilibrium framework; in addition, they measure welfare by expected consumer and producer surplus. This measure is quite unsuited to comparing the risks associated with different trade policies, which is our prime concern in the present paper.

3. Even when there are perfect equities markets, the assumption of identical consumers is crucial. Stiglitz (1981) has shown, for instance, that while the stock market equilibrium with multiplicative uncertainty and two commodities is always a constrained Pareto optimum with identical consumers, it essentially never is if consumers differ, either in their preferences for goods or their attitudes towards risk.
The paper is divided into three parts. In Part I we develop the basic model and derive the conditions under which free trade is Pareto inferior to no trade. In part II we use the simple model to analyze trade policy. In part III we show that the results are far more robust than the simple model developed in Parts I and II. We discuss the critical assumptions, present an alternative interpretation of the model, and suggest some interesting extensions to the analysis.

I. Comparison of Autarky and Free Trade

We construct a simple model with two regions which are identical, except that their output of a risky agricultural commodity is (perfectly) negatively correlated. There are, in each region, \( n \) identical farmers, and \( m \) identical consumers. We first describe the farmers, then the consumers, and finally the market equilibria with and without trade. We then compare the two equilibria and establish conditions under which free trade is Pareto inferior to no trade.

A. Producers

Each of the \( n \) identical producers owns one unit of land. A typical farmer allocates a fraction \( x \) of his land to growing a risky crop, which we denote by subscript \( r \), and the remainder, \( 1 - x \), to the safe crop, denoted by subscript \( s \). Output per acre of the risky crop is \( \theta \), a random variable, with mean unity and variance \( \sigma^2 \). The output of the safe crop is always unity. (These are just normalizations.)

Farmers are risk averse; they choose \( x \) before they know \( \theta \) (the weather) to maximize the expected utility of profits.\(^4\)

\(^4\) An inessential simplification of this analysis is that producers do not consume what they produce; their welfare depends only on what they obtain from selling their crops. See below, Section III.C.
(1) \( EU(\pi), U' > 0 \), \( U'' < 0 \),

where

(2) \( \pi = xp\theta + q(1 - x) \)

are the profits, \( p \) is the price of the risky crop, and \( q \) is the price of the safe crop.

Farmers are assumed to know the relationship between the price and \( \theta \), but since each farmer is small, he assumes he has no effect on price (in any state of nature).\(^5\) Our analysis thus focuses on competitive rational expectations equilibria. The expected utility maximizing choice of \( x \) is given by the solution to

(3) \( EU'(\pi)(p\theta - q) = 0 \)

or, if the variance of \( p \) and \( \theta \) are small,

(4) \( E\theta - q + \frac{U''}{U'} x E(p\theta - p\theta)^2 = 0 \).

Define \( A = -\frac{U''}{U'} \) as the coefficient of absolute risk aversion. If \( \text{var}(p\theta) > 0 \), then

(5) \( x \overset{\sim}{\sim} \frac{E\theta - q}{A \text{var}(p\theta)} \).

B. Consumers

It is convenient to represent the representative consumer by his indirect utility function

(6) \( V = V(I, p, q) \)

where \( I \) is income which is constant.\(^6\) For most of the analysis we shall make use of a special parameterization of the utility function. We let

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\(^5\) In our simple model, since the only source of price variability is supply variability, price will be a deterministic function of \( \theta \); in more general models, it would be a stochastic function.

\(^6\) \( I \) is the consumer's endowment of a third good (or a Hicksian composite commodity, representing all other goods), which is taken to be the numeraire.
(7a) \[ V = \frac{(1 - a)^{1-\rho}}{1 - \rho} \] for \( \rho \neq 1 \),

and

(7b) \[ V = \ln I - a \ln p - b \ln q \] for \( \rho = 1 \),

where \( \rho \) is the coefficient of relative risk aversion. These yield aggregate demand functions for the two commodities which have unitary price and income elasticities:

(8) \[ Q_r = \frac{aMI}{p}, \quad Q_s = \frac{bMI}{q} \]

where \( Q_i \) is aggregate demand for commodity \( i \), and \( MI \) is aggregate consumer income.

This specification of the utility function is chosen for several reasons: it greatly simplifies the calculations, it is the utility function for which consumer surplus calculations employed in conventional welfare analysis are valid;\(^7\) and it implies that there will be no income redistribution effects in the policy changes which we shall consider;\(^8\) and thus this specification enables a simple separation of the efficiency and distributional consequences of trade policy. Moreover, demand functions with unitary price elasticity play a critical borderline role in the analysis of risk with more than one commodity; if the elasticity of substitution between the two commodities is less than unity, the induced price variability results in farmers treating the safe commodity as if it were riskier than the risky commodity. (See Stiglitz [1972].) Finally the utility function exhibits constant (income) risk aversion,\(^9\) \( \rho \), and in the special case of unitary risk aversion of equation (7b)

\(^7\) See e.g. Samuelson (1942).

\(^8\) For other utility functions mean income of farmers will increase or decrease as a result of the opening of trade. See Section II.C. below.

\[ V_{Ip} = V_{Iq} = 0 \]

so changes in prices do not affect the marginal utility of income.

C. Market Equilibrium Without Trade

The structure of the equilibrium is simple:

(a) Producers decide on a crop allocation, \( x \), given their expectations about the relationship between price and \( \theta \). These expectations are assumed rational, that is, the expected relationship between price and \( \theta \) is in fact the one that emerges.

(b) After the crop is harvested, the crops are marketed competitively.

Market prices equate demand (given by eq. (8)) to supplies (which depend on \( x \) and \( \theta \)). If all farmers act alike then market clearing prices are

\[ p = \frac{ay}{x \theta} \]
\[ q = \frac{by}{1 - x} \]

where \( y = ml/n \), consumer expenditure per farmer.

In the present situation, (9) implies that the crop whose output is risky yields a perfectly safe return. If farmers are to grow both crops the returns must be identical

\[ p \theta = q \]

or, using (9),

\[ \frac{ay}{x} = \frac{by}{1 - x} \]

Hence the equilibrium solution is

\[ x = \frac{a}{a + b}, \quad q = (a + b)y \]

Producers' profits are given by \( \pi = q \) (using (10) and (2)) so producers'
welfare is simply \( U((a + b)y) \) using (11)). The representative consumer's average welfare in the logarithmic case is

\[
(12) \quad EV = V_0 + aE\ln \theta
\]

where \( V_0 \) is the utility without risk;

\[
(12a) \quad V_0 = \ln I - (a + b) \ln((a + b)y).
\]

The dollar value of the loss from the randomness in \( \theta \) is approximately \( \frac{a}{2} \sigma^2 \), or one half of the share of the risky crop in expenditure times the coefficient of variation in output. In the case of constant relative risk aversion not equalling unity

\[
(13) \quad EV = E\theta \frac{a(1-\rho)V_0}{1 - \rho}
\]

where

\[
(13a) \quad V_0 = \frac{n}{m(a+b)}^{(1-\rho)(a+b)} \frac{(1-\rho)(1-a-b)}{(1-\rho)}.
\]

The dollar value of the loss from risk is approximately

\[
\frac{1}{2}a(1 - a(1 - \rho))\sigma^2
\]

which agrees with the logarithmic case for \( \rho = 1 \).

Note that the unit elastic consumer demands transfer all the risk from farmers to consumers. Fluctuations in the supply of the risky crop affect only its price and not that of the safe crop. Except in this special case, production risk for one commodity will spill over to generate price risk for other commodities. (Stiglitz [1972].)
D. Free Trade

Now suppose that, to the east on the other side of a mountain range there is another region, identical to the one described above in every respect save one—the weather.  When it rains in the West it is dry in the East, and vice versa. The output of the risky crop is perfectly negatively correlated between the two regions. Formally, let

$$\theta^E + \theta^W = 2.$$  

Initially there is no trade between East and West, but there is a pass through the mountains which permits virtually costless exchange. If it is opened, there will be competitive free trade; if not, the regions will remain autarkic. Is free trade desirable?

Consider the symmetric equilibrium with free trade, in which each region plans the same fraction $x$ of the risky crop, $1 - x$ of the safe crop, yielding total supplies

$$Q_r = nx\theta + nx(2 - \theta) = 2nx, \quad 0 \leq \theta \leq 1,$$

and

$$Q_s = 2n(1 - x).$$

Prices will be perfectly stabilized at

$$(14) \quad \bar{p} = \frac{ay}{x}, \quad q = \frac{by}{1 - x}.$$  

Hence total profits are now

$$(15) \quad \pi = \bar{p}x\theta + (1 - x)q.$$  

Substituting (14),

$$\pi = (a\theta + b)y$$

and farmers' income is now risky.

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10 This rules out the conventional reasons for trade and allows us to concentrate on the risk aspect alone. Regions have a comparative advantage in weather alone in this model. Obviously, this heavily qualifies any policy conclusions which might be drawn from the study.
Farmers' equilibrium allocation is given by the solution to equation (16):

\[(16) \quad EU'(y(aθ + b))(aθy x - by 1-x) \equiv EM(θ;x) = 0.\]

E. Comparison of Free Trade and Autarky

Let us denote the equilibrium value of \( x, p, q, \) etc. with trade by \( \overline{x}, \overline{p}, \overline{q} \) and in autarky by \( \hat{x}, \hat{p}, \hat{q} \). Proposition 1 compares the allocations of the equilibria in the two situations:

**Proposition 1.** If farmers are risk averse, the safe crop is cheaper and the land area devoted to the risky crop smaller in free trade than autarky, i.e.

\[\overline{x} < \hat{x}, \quad \overline{q} < \hat{q}.\]

**Proof:** We know that

\[\overline{p} = \frac{ay}{x}.\]

Hence if \( \overline{x} = \hat{x} \), the return to farming the risky crop with trade constitutes a mean preserving spread in the return compared with what it was under autarky.

If the farmer is risk averse

\[\overline{Epθ} > q.\]

Hence, if the economy is stable (which, with our assumptions, it will be (see Appendix B)), since at \( x \), returns to the risky crop exceed those to the safe crop, \( x \) must increase when risk is eliminated. Hence \( \hat{x} > \overline{x} \), which in turn implies \( \overline{q} < \hat{q} \).

For small \( \sigma^2 \) a quantitative estimate of the adjustment can be made. From equation (5),

\[\overline{\frac{ay}{x} - \frac{by}{1-x}} \sim \frac{a^2y^2}{x^2} \sigma^2\]
or the trade allocation is

\[ x = \frac{a(1 - A\alpha y^2/a + b)}{a + b} \times \frac{a}{a + b} \left( 1 - \frac{ab}{(a+b)^2} \right) R^2 \]

where \( R \) is the coefficient of relative risk aversion for farmers

\[ R = -\frac{U''}{U'} \]

and \( \bar{R} \) is the value of \( R \) when \( \pi = \pi \).

Thus, using (11), the fall in \( x \) from moving to free trade is

\[ \frac{\Delta x}{x} = \frac{x - \bar{x}}{x} \approx \frac{ab}{(a+b)^2} \bar{R}^2. \]

The greater the risk aversion of farmers, and the larger the risk, the larger is the change in the allocation of crops.

More generally, we can easily establish

**Proposition 2.** (a) The more risk averse are farmers the greater is \( \Delta x \).

(Diamond and Stiglitz [1974]); (b) The greater the riskiness of \( \theta \) (in the sense of Rothschild-Stiglitz [1970]), the greater is \( \Delta x \), provided

(i) the range of \( \theta \) is not too great, or

(ii) there is constant relative risk aversion with \( R < 1 \),

(b) follows from the concavity in \( \theta \) of the expression \( M(\theta, x) \) in equation 11 (16).

**F. Welfare Analysis**

We can easily establish **Proposition 3.** Farmers are always worse off as a result of the opening of trade. In the new equilibrium mean income is

\[ M = U'[\theta + b] \left( ay \frac{x}{\theta} - by \frac{y}{1-x} \right) \]

then

\[ \frac{\partial M}{\partial \theta} = ayU'[\frac{1}{x} - \frac{x}{a\theta + b} \left( \frac{\theta}{x} - \frac{b}{1-x} \right)] \]

\[ \frac{\partial^2 M}{\partial \theta^2} = \frac{R}{x} U'' y \left[ - \frac{a\theta}{x} - b \left( \frac{2}{x} + \frac{1}{1-x} \right) + \left( R - \frac{R' \pi}{R} \right) \left( \frac{a\theta}{x} - \frac{b}{1-x} \right) \right] \]

If the range of \( \theta \) is small, \( \frac{a\theta}{x} - \frac{b}{1-x} \) will be small, so \( \frac{\partial^2 M}{\partial \theta^2} \) will be small. Alternatively, if \( R' = 0 \) and \( R < 1 \), then \( \frac{\partial^2 M}{\partial \theta^2} < 0 \). Clearly, these conditions can be weakened.
unchanged but risk is increased. A quantitative estimate of their welfare loss expressed as a percentage of mean income is provided by a Taylor Series approximation:

\[
\frac{\Delta U}{(a+b)yEU} = U\{(a + b)y\} - EU\{(a\theta + b)y\} \approx \frac{1}{2} \left( \frac{a}{a+b} \right)^2 R_0^2.
\]

Consumers now face no risk. This makes them better off but in return there is a change in \(x\), which always turns out to lower their welfare.

To see this, we ask what allocation \(x\) maximizes

\[
V = V \left[ \frac{-a}{x} \left( \frac{by}{1-x} \right) Z(\theta) \right]
\]

where \(Z(\theta) > 0\) depends on trade policy. Thus under autarky \(Z = \theta^a\), whilst in free trade \(Z = 1\).

However, for any \(Z(\theta) > 0\), we observe that the solution is

\[
x = \frac{a}{a+b} = \hat{x},
\]

the farmers' choice under autarky. Any other choice makes the consumer worse off. Hence, the change in consumer welfare in moving from autarky to free trade will depend on the magnitude of the resource shift, and on the consumer gain from the elimination of the risk he faces.

From Proposition (2), \(x\) depends on the size of risk, the importance of the risky crop in consumption, and the degree of farmers' risk aversion, while the consumers' benefit from risk reduction depends on the size of risk, the importance of the risky crop in consumption, and the degree of consumers' risk aversion.
In Appendix A we establish

Proposition 3. For given risk and demand parameters \((a, b)\), there exists a critical function \(f(\rho)\), \(f' > 0\) such that if \(R > f(\rho)\) free trade is Pareto inferior to autarky. Moreover, \(\lim_{R \to \infty} f^{-1}(R) < \infty\), \(\lim_{\rho \to 0} f(\rho) = 0\).

This critical function is illustrated in Figure 2, which shows that, for any given degree of risk aversion of consumers, there exists a sufficiently high degree of risk aversion for producers that both consumers and producers are worse off. For producer risk aversions slightly smaller than this critical value, producers are worse off, consumers better off, but the loss in welfare to producers exceeds the gain to consumers, in the sense that consumers could not bribe the producers to accept the opening up of trade by paying them a fixed sum.

II. Trade Policy

In this section, we consider the desirability of trade policy in our risky economy with incomplete markets. We establish four remarkable results:

Proposition 4. In the free trade equilibrium, some trade restriction is Pareto improving, provided producers are sufficiently risk averse.

Proposition 5. In the no trade equilibrium, allowing some trade will always be a Pareto improvement.

Proposition 6. Near the free trade equilibrium, quotas are preferable to tariffs, again provided producers are sufficiently risk averse.

Proposition 7. Near the no trade equilibrium, tariffs are preferable to quotas.

The intuition behind these results is simple. In the free trade equilibrium, consumers face constant prices, while producers face considerable risk. A slight trade restriction (with revenues (if any) redistributed to producers) leaves producers' mean income unchanged, but reduces their risk. They are thus better off. They reallocate their crops, and this reallocation effect makes consumers better off; since consumers initially face no risk, there is a negligible cost associated with the slight increase in risk which they face. Thus the reallocation effect dominates the risk effect. Since quotas are most
Figure 2. Determinants of inferiority of free trade
effective in the states where output (and hence income) is very high or very low, increasing income in the latter case and decreasing it in the former, they are more effective in reducing the risks faced by producers than are tariffs. (How we compare alternative trade policies is discussed in Section A.) The argument for the effect of trade policy at the no trade equilibrium follows along similar lines.

A. Symmetric Trade Policies

We assume each country pursues identical trade policies, so that symmetry is not disturbed. We also assume throughout that any revenues from trade policy are distributed as lump sum payments to producers. Suppose that when a policy instrument is set at level \( z \), in state \( \theta \) (so that production of the risky crop is \( x_\theta \)) the level of imports is \( x(\theta, z) \).

Symmetry then requires that

\[
(26) \quad t(\theta, z) = -t(2-\theta, z).
\]

If increases in \( z \) correspond to liberalizing trade policy, and if the effect of the liberalization is to increase trade in every state of the world (or not decrease it), then

\[
(26') \quad \frac{dt(\theta, z)}{dz} = -\frac{dt(2-\theta, z)}{dz} > 0 \text{ for } 0 \leq \theta \leq 1.
\]

Symmetry also implies that

\[
(26'') \quad E_t(\theta, z) = 0 \quad \text{and} \quad E_t \frac{dt}{dz} = 0.
\]

For example, with a quota at level \( T_x \),
\[ t(\theta, T) = \begin{cases} 
T & \theta \leq 1 - T \\
1 - \theta & 1 - T \leq \theta \leq 1 + T \\
-T & \theta \geq 1 + T 
\end{cases} \]

The corresponding distribution of domestic prices (in the symmetric case) will be

\[ p(\theta, x^*, z) = \frac{ay}{x^*(\theta + t(\theta, z))} . \]

In our example of quotas, the domestic price will be

\[ p = \begin{cases} 
\frac{ay}{x^*} & \theta \leq 1 - T \\
\frac{ay}{\theta + T} & 1 - T \leq \theta \leq 1 + T \\
\frac{ay}{x^*} & \theta \geq 1 + T 
\end{cases} \]

With quotas, either there is free trade (in the central range of values of \( \theta \)) or trade is set equal to the level of the quota in the importing country, and domestic prices then clear the market given the constrained supply.

The corresponding trade policy with tariffs is to impose a duty at rate \( \tau \) on imports, and a tax on exports at the same rate.\(^{12}\) Tariffs operate quite differently from quotas, for over the middle range of values of \( \theta \), prices in each country will differ by less than the height of the tariff barrier, and no trade will occur. If trade does take place, it will move domestic prices to within \( 2\tau \) of each other. In the event of positive trade occurring, the form of the function \( t(\theta, \tau) \) can be found from the equation of the domestic price differences:

\(^{12}\) It would make no difference to the argument if there were a tariff at rate \( 2\tau \) on imports and no tax on exports. What matters is the difference in prices in the two countries.
\[ p^E - p^W = \frac{\alpha \nu}{x} \left[ \frac{1}{\hat{\theta} + t(\theta, \tau)} - \frac{1}{2 - \theta - t(\theta, \tau)} \right] = 2\tau \]

There will be no trade for \( \hat{\theta} \leq \theta \leq 2 - \hat{\theta} \), where \( \hat{\theta} \) is the value of \( \theta \) for which \( t = 0 \) in this equation, namely

\[ \hat{\theta} = 1 - \frac{1}{2} \left( \frac{1}{\sqrt{k + 4} - k} \right) \quad \text{where} \quad k = \frac{\alpha \nu}{\tau x}. \]

The value of \( t(\theta, \tau) \) is then

\[
\begin{align*}
(29') \quad t(\theta, \tau) &= \begin{cases} 
\hat{\theta} - \theta & 0 \leq \theta \\
0 & 0 \leq \theta \leq 2 - \hat{\theta} \\
-\hat{\theta} + (2 - \theta) & \theta \geq 2 - \hat{\theta}
\end{cases}
\end{align*}
\]

In the absence of uncertainty, such tariffs and quotas would have identical effects, but it is clear that in the presence of uncertainty they have quite different effects on the distribution of prices, profits and consumption levels. The differences between the price, consumption, and profit distributions under quotas and tariffs are shown in Figures 3-5. Consumption of the risky crop in state \( \theta \) with policy \( t(\theta, z) \) is \( x(\theta + t(\theta, z)) \).

\[ \text{B. Effect of Trade Policy on Consumers} \]

In equilibrium producers must receive the amount spent by consumers on the safe and risky crops, less the value of consumers' purchases of the risky crop from the other country, so producer profits will be

\[ \pi(\theta, z) = (a + b)y - D(\theta, z) \]

where \( D(\theta, z) \) is the trade deficit; its value will depend on the world price, which in turn will depend on the specification of trade policy. For example, with quotas it will depend on how the rents generated by the quotas are allocated between exporting and importing countries. Fortunately, it is not necessary to be specific about the determination of \( D(\theta, z) \) since we can use the symmetry assumption, namely that
Figure 3. Price distribution
Figure 4. Consumption distributions

Consumption $x(\theta + t(\theta, z))$

- Autarky
- Tariffs
- Quotas
- Free trade

$1 - \theta$ $1$ $1 + T$ $\theta$
Figure 5. Profit distributions
\[(31) \quad D(\theta, z) = -D(2-\theta, z), \quad D(\theta, z) \geq 0 \text{ as } \theta \leq 1.\]

We follow the convention that \(z = 0\) corresponds to autarky, \(z = 1\) to free trade; then

\[(32) \quad ED(\theta, z) = 0, \quad \text{all } z, \quad \frac{dD}{dz} \geq 0 \text{ as } \theta \leq 1.\]

It is immediate from (32) that any opening of trade represents a mean-preserving spread in farmers' profits compared to autarky, and so they must be worse off.

Moreover, an infinitesimal trade liberalization has an infinitesimal effect on risk, which, starting at zero risk, is costless, although any finite liberalization makes farmers worse off. This is shown in Fig. 1, where farmer's utility is flat at autarky.

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13 The effect of a small change in policy can be found by differentiating totally with respect to \(z\), where \(z = 0\) corresponds to autarky (\(t=0\)), and \(z = 1\) to free trade (\(t=1-\theta\)). The effect of the change of producers' welfare is, from equation (30):

\[
\frac{dEU(\pi)}{dz} = \frac{\partial x}{\partial z} \frac{\partial EU(\pi)}{\partial x} - EU'(\pi \ast) \frac{dD(\theta, z)}{dz}.
\]

The first term is zero by the envelope theorem, whilst the second can be written

\[
\frac{dEU(\pi)}{dz} = -\frac{1}{\int [U'(\pi \ast(\theta, z)) \frac{dD(\theta, z)}{dz} + U'(\pi \ast(2-\theta, z)) \frac{dD(2-\theta, z)}{dz}]}dF(\theta)
\]

where \(F(\theta)\) is the distribution function of \(\theta\), and we have used the symmetry of \(F\) about the mean \(\theta = 1\). From equation (31) this can be written

\[
\frac{dEU(\pi)}{dz} = -\frac{1}{\int [U'(\pi \ast(\theta, z)) - U'(\pi \ast(2-\theta, z))]} \frac{dD(\theta, z)}{dz} dF(\theta).
\]

By the Mean Value Theorem, there is a \(\lambda\), \(-1 < \lambda < 1\), such that

\[
\frac{dEU(\pi)}{dz} = \begin{cases}
0 & \text{if } t = 0 \\
\frac{1}{2} \int D(\theta, z)U''(a+b)y + \lambda D(\theta, z) \frac{dD(\theta, z)}{dz} dF(\theta) & \text{if } t > 0.
\end{cases}
\]

Thus, an infinitesimal liberalization has no effect on producers, but any finite liberalization makes them worse off if they are risk averse.
The fact that producers become unambiguously worse off as trade becomes more liberalized does not necessarily mean that they increasingly shift production to the safe crop. Their crop allocation decisions depend on expected marginal utility, not expected utility, and, as Newbery-Stiglitz [1980] have shown in other contexts, there may be no simple relationship between changes in the two. To see what happens as we change trade policy we proceed in two steps. Suppose a producer was initially in equilibrium at $x^*$ where his expected utility is maximized, as in Fig. 6a. Trade liberalization lowers utility as shown, and we are interested in what happens to the expected marginal return to growing the risky crop relative to the safe crop, holding $x$ constant at $x^*$. In Fig. 6a it falls, as the slope at $B$ is negative, whilst at $A$ it was 0 with both crops equally attractive. This will tend to shift farmers out of the risky crop and drive up its price, changing its attractiveness. However, if the equilibrium is to be stable, it must be the case that, for given trade policy $z$, the net marginal return to growing the risky crop must decrease as $x$ increases, as shown in Figure 6b. In Appendix B we discuss analytically the conditions required for stability; but if the equilibrium is stable as shown, then a necessary and sufficient condition for trade liberalization to reduce the supply of the risky crop (i.e. $dx/dz < 0$) is for trade policy to lower the marginal net return to growing the risky crop, holding $x$ constant.

\begin{equation}
\frac{dx}{dz} \leq 0 \quad \text{as} \quad \frac{\partial}{\partial z} \text{EU}'(\pi)(r-q) \leq 0
\end{equation}

where $\pi$ is given by equation (30) and, from (28)

\[ r = p\theta = \frac{av}{x} \frac{\theta}{(\theta + t)}. \]
Farmer's expected utility $EU(\pi)$

**Figure 6a**

$x^*$

$x$, land allocated to risky crop
Figure 6b. General equilibrium response to policy, stable case.
In Appendix B we show that
\[ \frac{dx}{dz} \leq 0 \quad \text{as} \quad R > 1 + \frac{b}{a}. \]

C. Effect of Trade Policy on Consumers

The effect on consumers is described by

\[ \frac{dEV}{dz} = \frac{\partial EV}{\partial z} + \frac{\partial EV}{\partial x} \frac{dx}{dz} \]
\[ = EV \frac{\partial p}{\partial z} + E \left( V_p \frac{\partial p}{\partial x} + \frac{\partial q}{\partial x} \right) \frac{dx}{dz} \]

Since
\[ p = \frac{ay}{x(\theta+t(\theta,z))}, \quad q = \frac{by}{1-x} \]

\[ \frac{\partial p}{\partial z} = -\frac{p}{(\theta+t)} \frac{dt}{dz} \]

\[ \frac{\partial p}{\partial x} = -\frac{p}{x}, \quad \frac{\partial q}{\partial x} = \frac{q}{1-x} \]

and, from Roy's identity

\[ \frac{V_p}{p} = -V_I x(\theta+t), \quad \frac{V_q}{q} = -V_I (1-x). \]

Hence

\[ \frac{dEV}{dz} = E V_I \left\{ p \frac{dt}{dz} + (p(\theta+t) - q) \frac{dx}{dz} \right\}. \]

Finally, from equation (26), we know that \( \frac{dt}{dz} \) is negatively correlated with \( \theta \). We can now examine special cases.

(a) Autarky At \( z = 0, \ t = 0, \ p\theta = q \), so

\[ \frac{dEV}{dz} = x E V_I p \frac{dt}{dz}. \]

This will be positive if \( V_I p \) is positively correlated with \( p \),

(which is negatively correlated with \( \theta \)), i.e.
(41) \[
\frac{dEV}{dz} > 0 \quad \text{if} \quad \frac{V_I}{V_I} p \frac{1}{p} = a(p-1) < 1 .
\]

The effect of trade liberalization from the consumer's point of view is that of a mean quantity preserving decrease in the riskiness of consumption, which, if consumers are risk averse, that is, utility is concave in the risky good, is beneficial.

(b) Free Trade. Free trade corresponds to the trade policy 
\[
t(\theta, z) = 1 - \theta .
\]

For consumers, who face no risk in free trade,
\[
\frac{dEV}{dz} \bigg|_{t=1-\theta} = V_I E \left( \frac{p}{p} x \frac{dt}{dz} + (p - q) \frac{dx}{dz} \right) .
\]

Since \( E \frac{dt}{dz} = 0 \) (by symmetry)

(42) \[
\frac{dEV}{dz} = (p - q) V_I \frac{dx}{dz} .
\]

This has the same sign as \( \frac{dx}{dz} \), since, for risk averse farmers, \( \frac{p}{p} \) must exceed \( q \) to induce them to plant the risky crop.

Using the result of Section II, we see that if \( R > 1 + b/a \), there is an intermediate level of trade, between no trade and free trade, at which consumers welfare is maximized, as illustrated in Figure 1.

The figure makes clear that at the autarky point, allowing an infinitesimal amount of trade improves consumer welfare, but has no effect on producers' welfare; while at \( z = 1 \) (free trade) there is a considerable range of restrictions on trade which make both producers and consumers better off.

D. Quotas Versus Tariffs

Quotas and tariffs are well known to be equivalent in the absence of uncertainty, but with uncertainty, they have very different implications,
as we have already noted. We would like to make statements like "a tariff is preferable to an 'equivalent' quota if ______ is true" but to do this, we have to define 'equivalent'. Any such comparison involves a certain degree of arbitrariness. One approach would be to analyze the optimal policy, which would involve mixtures of tariffs, quotas, and perhaps other forms of trade restrictions. Here, we take a somewhat simpler approach. We focus on trade policies which yield zero revenue, i.e., the quota is given on a pro-rata basis to producers, and the tariff is matched by a lump sum transfer to producers so that the government gets no revenue from the price distortion. The simplest comparison is between trade policies having the same effect on trade, i.e., letting \( x_t(\theta,z) \) be the amount of trade occurring when output is \( x_\theta \) for trade policy \( z \), then we compare policies for which \( |E t(\theta,z)x| \) is the same.

Starting initially at no trade, it is immediate that all small trade policies in which revenues are redistributed back to producers are equivalent from the point of view of the producer; all have zero effect on his welfare and on his action. For consumers, however, there is a difference, which can best be seen by examining the effect on expected utility using the direct utility function. Since for all such programs, \( x^* \) is the same, all such programs represent mean preserving changes in the distribution of the consumption of the risky crop, \( Q_r \). Utility is a concave function of \( Q_r \). We noted earlier that tariffs lead to less trade in states of nature near the mean (since in those states, the quota is not binding, the solution is equivalent to free trade for those states). Thus, there is more trade in states of nature far from
the mean, i.e. more exports in high θ states and more imports in low states. Thus $Q_r$ is less disperse with tariffs as shown in Figure 4. We have thus established for trade restrictions which allow only a small amount of trade, tariffs are Pareto superior to quotas.

At the other extreme, beginning from a position of free trade, consumers' ranking of trade policies is based simply on their effect on $x$, since the risk effect, at given $x$, is zero.

Clearly, producers have the same mean income under each trade policy, but quotas change the price only in the tails of the distribution as shown in Figure 3; thus, quotas result in a less disperse distribution of profits, and hence producers' utility is higher with quotas than with tariffs.

The effect on their action follows the same line as the argument used to sign equation (33). Since a quota reduces producers' risk more effectively than a tariff, it will cause a larger increase in the supply of the risky crop than a tariff, provided reducing risk leads to an increase in the risky crop at all. The condition for this was found in Appendix B to be that farmers be sufficiently risk averse, $R > 1 + b/a$. In this case the increased allocation to the risky crop will make consumers better off.

Hence, we have established that near the free trade equilibrium a quota has a greater allocative effect, a greater improvement on producer welfare, and a greater improvement on consumer welfare than does the tariff of corresponding restrictiveness on trade, provided that producers' risk aversion is large enough.
III. Extensions

A. Multiple Equilibria and Stability

In this paper we have focused on the symmetric equilibrium. There may also exist asymmetric equilibria, where farmers in one country allocate a fraction, $x_1^*$ to the risky crop, and farmers in the other country allocate a fraction $x_2^*$ to the risky crop, $x_1^* \neq x_2^*$. For each value of $x_1$, we can define the equilibrium response of farmers in country 2 and conversely.

\[(43) \quad x_1 = g_1(x_2), \quad x_2 = g_2(x_1).\]

An equilibrium is any pair $(x_1^*, x_2^*)$ satisfying (43). Figures 7 and 8 illustrate situations where the reaction functions intersect several times. There is a natural dynamics associated with these reaction functions. Figure 7 illustrates a situation where the symmetric equilibrium is stable, while the symmetric equilibrium in Figure 8 is unstable. If farmers are risk neutral, it can be shown that the symmetric equilibrium is unique and stable; if farmers are infinitely risk averse, there are multiple equilibria, and the symmetric equilibrium is unstable. (See Newbery and Stiglitz [1981b].)

B. Financial Markets

This paper so far has investigated the consequences of free trade in commodities when there is no securities market. We now wish to consider how these results are affected by the availability of various financial instruments. First, we note that in the central case of consumers having logarithmic utility functions, there is no scope for future markets in the no trade situation (since price is not variable in the free trade situation there is never scope for a futures market there); for then $V_{ip} = 0$, the marginal utility of income is the same independent of the state of nature, and hence market equilibrium requires that if a small futures market would be introduced, it be actuarially fair. But for producers, profits are constant, and they too would require an
Figure 7. **Five equilibria, with stable symmetric equilibrium, (two boundary equilibria stable)**
Figure 8. Three equilibria, with symmetric equilibrium unstable
actuarially fair futures market. Hence the only equilibrium is one where there is no trade on the futures market.

The consequences of opening up a market for ownership shares in farms is more serious. (We ignore all the problems of moral hazard, which are fundamental to an understanding of why such markets might not exist.) If farmers could purchase shares in each others' farms, then income of farmers would be constant; they would thus be indifferent between the opening of trade and autarky, but consumers would be unambiguously better off. Thus, the standard result that free trade is preferable to no trade is restored, if there is a complete set of securities markets.

But an essential part of the argument so far is that trade is costless. With costly trade, the possibility that opening trade will at least lower the welfare of producers is restored. With free trade in securities markets, farmers' income is smoothed, but the opening of trade leads to farmers' net income being lowered by the expenditure on transport costs. Thus farmers are worse off as a result of the opening of trade.

C. Robustness of Model

In the analysis, we have employed a number of special assumptions. Our objective was to formulate the simplest model illustrating the points at issue, not the most general. We have considered a number of modifications of the model. These lead us to the conclusion that the model is in fact very robust. Changing any of the assumptions obviously changes the precise conditions under which a Pareto inferior trade equilibrium will result, but the qualitative results remain. It is still true that (i) producers may be
worse off; (ii) consumers may also be worse off, or, if better off, their gains will be insufficient to compensate the producers; and (iii) there exists some form of trade intervention which would be a Pareto improvement. In this section, we discuss briefly a few of the directions in which the model may be extended.

(a) Alternative Specifications of Consumer Demand Curves. The major reason that the unit price elasticity assumption is employed is that it avoids the confusion between transfer and risk effects. If the elasticity of demand is not unity, a mean quantity preserving change in the price distribution will change mean income of farmers. (Newbery-Stiglitz [1981b].) In particular, if demand curves have constant elasticity greater than unity, farmers' revenue is a convex function of consumption, so, at each value of \(x\), mean income is higher under autarky than with free trade. Although in autarky, producers' income is variable, it is less variable than output, and hence less variable than income under free trade. It can thus be shown that (i) a slight increase in trade from the no trade position always increases producers' welfare; (ii) provided the elasticity is not too large, a slight restriction on trade from the free trade position also increases producers' welfare. (See Figure 9a.)

The effect on consumers is more difficult to analyze. There are now three effects: (i) the risk effect; as before, opening trade reduces consumer risk; (ii) the transfer effect; because of the change in the distribution of prices, mean expenditure of consumers is increased, which makes them worse off; (iii) the allocation effect; because of the transfer effect, the allocation effect will be smaller, and may actually be positive rather than negative. Figure 9b illustrates the effect that this has on consumer utility as a
Figure 9a. Effect of Trade on Producers
Figure 9b. Effect of trade on consumers
function of the level of trade.

Similarly, if the elasticity of demand is less than unity the reduction in price variability reduces producers' mean income, and, if the elasticity is greater than 0.5, opening trade increases the variability of their income. Thus, producers are unambiguously worse off. Again, the effects on consumers are more complicated: now the transfer effect is positive, but the allocation effect is larger. Thus there are a large variety of possible patterns.

If the price elasticity is very small (less than 0.5), then pre-trade income variability is greater than post-trade income variability, in which case the opening of trade may be Pareto superior even without direct compensation schemes.

For elasticities of demand near unity, however, the qualitative properties remain unaffected: free trade is Pareto inferior to no trade, provided farmers' are sufficiently risk averse; if the elasticity is greater than unity, a slight liberalization of trade from the no trade position is Pareto improving; if the elasticity is less than unity, producers are strictly worse off; a slight restriction on trade from the free trade position is Pareto improving provided producers are sufficiently risk averse.

(b) Alternative Specifications of the Risk. We assumed that the two regions had perfectly negatively correlated crops. The results, however, remain valid so long as the correlation is not +1. For any positive degree of correlation (less than perfect), farmers' mean income will (with the unitary price elasticity assumption) remain unchanged, while the opening of trade will induce some degree of income variability. Note that the case of perfect positive correlation (which has received extensive discussion within the trade literature) is an extreme and special case. This is discussed further in Newbery and Stiglitz (1981a).
(c) Alternative Specification of Producers' Utility Functions. Throughout we assumed that producers only consumed the numeraire good; the relative price of the agricultural good did not affect their welfare, except in so far as it affected their income.

We could easily extend the model to allow producers to consume the goods which they produce. All that is critical for our analysis is that, for all values of the price, they remain producers, i.e. they continue to sell the agricultural goods (which, given our assumption that they have no other sources of income, will remain the case provided variations in the output of the risky agricultural good are not too large). The reason that introducing producers' consumption complicates the analysis is that net sales (to pure consumers) are then not just equal to (or in general a simple function of) the gross output. If, however, we assume the producers also have a logarithmic utility function (or an ordinal transformation of a logarithmic utility function), the analysis is completely unaffected: producers' expenditures on each of the crops are a constant fraction of gross income, and although the details of the calculations need to be modified, all the qualitative properties remain unchanged.

The one case in which opening of trade always is Pareto superior is that where everyone within each of the two regions is identical (i.e. there is no distinction between producers and consumers). If all individuals are identical, then, before trade, they consume precisely what they produce; the price distribution is of no concern. Since after opening up trade, they could still just consume what they produce, opening up trade must make them better off.
(d) **Two Factors and Comparative Advantage.** Specialists in traditional trade theory have, on a number of occasions, expressed an uneasiness with our analysis, on the grounds that it appears to ignore the principle of comparative advantage which underlies conventional trade theory. Note that this is not quite correct. Although there is no long run comparative advantage, each of the countries having precisely the same endowments and tastes, every period there is a significant comparative advantage, depending simply on the weather.

We can, however, easily incorporate elements of traditional comparative advantage theory within our framework. Although we leave the formal development of such a synthesis to another occasion, let us briefly show how it may be done. Assume, for simplicity, that there are two factors, say capital and labor. Assume, that the safe crop is labor intensive. For simplicity, let us assume that the two countries have the same factor endowments. Clearly, there is again no "long run" comparative advantage. We can ask, however, what happens as a result of the opening up of trade. Our previous analysis applies almost identically, except now farmers do not face a straight line production possibilities schedule, but face a concave production possibilities schedule. It is still true, however, that the opening up of trade will lead to an increase in the production of the safe crop, and this in turn implies that the opening up of trade will lead to an increase in the return of the factor which is intensive in the safe crop (in this case labor). Assume, now, that the two countries have slightly different factor endowments. There will now be two effects: in the absence of risk, the opening up of trade will lead to the equalization of factor prices, a decrease in the price
of labor in one country, and an increase in its price in the other. Now, however, there is an additional risk effect, which will tend to shift production toward the safe crop (more, presumably, in the country which has a comparative advantage in its production). This effect may outweigh the first effect, so that, still, the price of labor in both countries rises; it is even possible that factor price differentials widen.

D. The Critical Assumptions

In the preceding section, we showed that the results of our analysis are far more general than the model employed in Parts I and II, in which we made the assumptions of unitary price and income elasticities. There are, however, three critical aspects of our analysis (which distinguish it from most of the earlier literature on trade and uncertainty): (a) the price distribution is determined endogenously, as a result of the decisions of producers in both countries; the opening of trade affects this price distribution in a critical way; (b) there is no stock market (or insurance market) by which farmers in one country can completely diversify out of their risks by buying shares in farmers in the other country; and (c) the risks in the two countries are not perfectly correlated. All three of these are important (and we would argue, plausible assumptions). Although there may be disagreements about the precise reason why there is not a perfect set of risk markets, the fact of the matter is that producers cannot perfectly insure the risks which they face. The assumption that there are no risk markets would appear to be a far better approximation to reality (at least for the situation examined here) than the contrary assumptions, conventionally employed, that such markets are perfect.

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14 This is equivalent to requiring the direct utility function be concave in the risky product. So long as producers cannot completely diversify out of the risks they face, the possibility of Pareto inferior trade (as well as the other qualitative properties discussed in the paper) remain.

15 As we argued, so long as the returns are not perfectly correlated, there is a possibility of Pareto inferior trade, although the precise conditions clearly depend on the correlation between the two countries' output.
E. Alternative Interpretations

The model we have formulated can be looked at in quite a different way: what would be the consequences of establishing a buffer stock which stabilized commodity prices. For a "perfect buffer stock" would stabilize producer prices, thus having precisely the same effect on producers as the opening of trade. 16 Thus, a buffer stock scheme may be Pareto inferior to no stabilization. Indeed, the analysis shows that, in the absence of a complete set of securities markets, the market may generate an excessive amount of intertemporal arbitrage, and the amount of price stabilization may be greater than is Pareto optimal. 17

Finally, note that in our model there is trade without comparative advantage (except in the sense noted above that, at any particular date, one country has a larger supply of the risky crop than the other). The direction of trade varies from year to year, and does not depend on factor endowments or tastes, but simply on the outcome of some random variable. Since there is considerable evidence that much trade cannot be explained solely on the basis of factor endowments or tastes, it seems desirable to explore models in which there is trade without comparative advantage. Of course, there are other possible models, emphasizing economies of scale and monopolistic competition, and again, in such models, opening trade need not be Pareto improving.

IV. Concluding Remarks

There is an argument, popular with economists, that if markets are competitive and agents well informed, then government intervention will lead to inefficiency. This argument has been used both to argue against

16 Obviously, a finite buffer stock cannot indefinitely perfectly stabilize price, and so the analogy with perfect price stabilization must be used cautiously.

17 For a more extensive discussion of this, see Newbery and Stiglitz (1981a, 1981b).
restrictions on trade and against the establishment of international buffer stocks. At the moment, many LDCs are pressing for the establishment of international buffer stocks for certain core primary commodities. However, most commodity markets are very competitive, forward markets exist to disseminate information quickly through the market, and private agents can stockpile if it is privately profitable. At best, it is argued, the scheme will provide an inefficient mechanism for a concealed aid transfer, and would be better replaced by an international credit agency (such as the IMF) and direct aid transfers.

Our analysis questions the premises of this reasoning. We show that there is no presumption that free markets will be Pareto optimal. But our analysis is not simply negative. We provide a framework which enables, in any situation, a detailed evaluation to be made of whether trade liberalization or a commodity buffer stock will be a Pareto improvement. Our analysis suggests that, under quite plausible conditions, it is desirable for the government not only not to introduce a buffer stock, but actually to restrict the arbitrage activities of speculators. Our analysis also shows that some trade restrictions may be desirable in the presence of imperfectly correlated risk.

Our analysis has, quite obviously, left out all considerations of the political processes by which trade policy becomes determined. Though the greater effectiveness of quotas in reducing producer uncertainty, which we established in section II, may provide part of the explanation of the seeming preference of many trade associations for quotas over tariffs, there is little to suggest that present trade barriers are chosen with the considerations we have raised in mind. What our analysis does explain, however, is why the simplistic arguments for free trade, making use of the competitive paradigm, with all of its assumptions, including that of a complete set of risk markets, are often far from persuasive.
Appendix A.
Proof of Proposition 3

We continue the convention that carets refer to autarky, double bars to free trade. Then from (24), we write (for some constant $K$)

\[ \hat{E}V = KE[\hat{x}^a (1-\hat{x})^b \theta^a]^{1-\rho} \]

\[ \bar{V} = K[(\bar{x})^a (1-\bar{x})^b]^{1-\rho}. \]

The equation $R = f(\rho)$ is defined implicitly by $\bar{V} = \hat{E}V$, or

(A.1) \[ [E\theta^a(1-\rho)]^{1/(1-\rho)a} = \left(\frac{\bar{x}}{\hat{x}}\right)^{\frac{(1-\bar{x})}{b/a}}. \]

It is immediate that the LHS of (A.1) is monotonically decreasing function of $\rho$, with

(A.2) \[ \lim_{\rho \to \infty} [E\theta^a(1-\rho)]^{1/(1-\rho)a} = \theta_{\text{min}} < 1 \]

(A.3) \[ \lim_{\rho \to 0} [E\theta^a(1-\rho)]^{1/(1-\rho)a} = E\theta = 1. \]

Since $\hat{x} = \frac{a}{a+b}$, the RHS is a single peaked function of $\bar{x}$ with the maximum attained at $\bar{x} = \hat{x}$, as noted above in equation (25). Hence, using Proposition 2, the right hand side is a monotonically decreasing function of $R$.

To prove the theorem all we need to show is that if $R = \infty$ there exists a solution to equation (A.1) for some values of $\rho < \infty$. At $R = \infty$, farmers are only concerned with their worst state. We can thus show that the RHS of equation (A.1) equals \[ \theta_{\text{min}} \frac{(1+c)}{(\theta_{\text{min}} + c)^{1+c}} \equiv h(c, \theta_{\text{min}}) \]

where $c = b/a$. By differentiation, $h$ is monotonically decreasing in $c$, and

\[ h(0, \theta_{\text{min}}) = 1, \]

\[ \lim_{c \to \infty} h(c, \theta_{\text{min}}) = \theta_{\text{min}} c^{-\theta_{\text{min}}}, \]

so, for $0 < c < \infty$

\[ \theta_{\text{min}} < c^{1-\theta_{\text{min}}} \theta_{\text{min}} < h(c, \theta_{\text{min}}) < 1. \]
But this together with (A.2) and (A.3) implies there exists some $\rho$ for which equation (A.1) holds.

Moreover, as $\rho \to 0$, the gain from risk reduction by consumers goes to zero. Hence the reallocation effect must go to zero. Hence $f(\rho) \to 0$.

The exact location of the curve $f(\rho)$ depends obviously on $a$, $b$ and the magnitude of risk. We have not yet been able to obtain a general characterization of how each affects the boundary curve. For instance, an increase in risk decreases both the left and right hand sides of equation (A.1) (the reallocation effect is larger, but the reduction in risk borne by consumers is also larger.)
Appendix B.

Conditions for Trade Liberalization to Reduce Land Allocated to the Risky Crop

We derive conditions on the assumption that the equilibrium is stable, which, however, is not always ensured by the usual second order conditions. For stability in a closed economy, we require

\[(Bl) \quad EU''(p\theta - q)^2 + EU''[\frac{3p}{3x} x\theta + (1 - x)\frac{3q}{3x}](p\theta - q) + EU'(\frac{3p}{3x})\theta - \frac{3q}{3x} < 0\]

The second order condition only ensures the negativity of the first term. However, using (37), we can rewrite (Bl) as

\[- EU'(\frac{p\theta}{x} + \frac{q}{1-x}) < 0\]

which is obviously satisfied. However, in an open economy, stability is a more complicated matter, as in Section IIIA.

From equation (33) we seek conditions for

\[\frac{\partial}{\partial z} EU'(\pi)(r-q) < 0\]

i.e.

\[EU'(\pi) \frac{\partial r}{\partial t} - (r-q)EU''(\pi) \frac{\partial D}{\partial t} \frac{dt}{dz} < 0\]

But

\[\frac{\partial r}{\partial t} = - \frac{ay}{x} \frac{\theta}{(\theta+t)^2}\]

At autarky, \(z = 0\), \(t = 0\), and \(r = q\), so the condition is

\[- U'E \frac{1}{\theta} \frac{dt}{dz} < 0\]

which is guaranteed by the negative correlation of \(\theta\) and \(dt/dz\) (see equation (26)).

At free trade, \(z = 1\), \(t = 1 - \theta\), and the condition is
\[ E - \left( \frac{ay}{x} - (r-q)U''(\pi) \frac{\partial D}{\partial z} \right) \frac{dt}{dz} < 0. \]

For small variances, \( r \approx q \) and the second term is small relative to the first, so the condition becomes

\[ EU'\theta \frac{dt}{dz} > 0. \]

Since \( \theta \) and \( \frac{dt}{dz} \) are negatively correlated, we require \( U'\theta \) and \( \theta \) to be negatively correlated for the result.

At free trade, \( \pi = ay + by \), and

\[ \frac{dEU'}{d\theta} = U' + U''ay\theta \approx U'(1- \frac{Ra}{a+b}) < 0 \]

Thus

\[ \frac{dx}{dz} \leq 0 \quad \text{as} \quad R \geq 1 + \frac{b}{a} \]

and the risky crop is reduced with trade liberalization if farmers are sufficiently risk averse.
REFERENCES


