THE DETERMINANTS OF MEMBER-BANK BORROWING: AN ECONOMETRIC STUDY

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ABSTRACT

This paper presents a formal model of commercial-bank borrowing from the Federal Reserve System which integrates profit, need, and surveillance considerations. We first develop a static model. Dynamic factors are then introduced and are shown to imply a distributed-lag reaction on the part of the banks.

Empirical evidence in support of the model is presented. Multiple regression equations are estimated with weekly data for several categories of member banks. The results indicate that member-bank borrowing can be successfully explained by the factors put forth in the paper. We also find significant behavioral differences among the various classes of banks.
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This paper presents a partial-equilibrium theory of member-bank borrowing from the Federal Reserve System. This theory, which derives from the work of Polakoff, Turner, and Meigs, portrays bank borrowing as primarily a short-run phenomenon. Faced with an immediate need for additional reserves, banks must weigh the costs of reserve adjustment against the disutility of increased debt to the Federal Reserve. The formulation adopted allows us to unite and to parameterize the central forces of various competing theories: namely those of banker profit, need, and reluctance, and of Federal Reserve surveillance.

Relying primarily on multiple regression, the empirical section of the paper tests the theory against weekly data covering the period July, 1953 through December, 1963. Because the theory describes decision making at the individual bank level, we use the most disaggregate figures available: specifically, aggregate borrowing and reserve flows for each

of the four main categories of member banks (New York City, Chicago, other reserve city, and country) for which information is regularly collected. Our results affirm both the broad outlines of the theory and the value of disaggregation. At the same time, they provide evidence that the availability of unborrowed reserves and elements of distributed-lag adjustment are somewhat more important determinants of borrowing than current differentials between the discount rate and market yields on Treasury bills. Additional evidence suggests that member-bank reluctance to borrow and speeds of adjustment vary with class of bank.

I. THE STATIC THEORY OF MEMBER-BANK BORROWING

A. The Free-Market Model

We begin with a number of simplifications. First, we assume that at all times banks remain fully loaned up, investing their loanable funds in marketable securities. For convenience, we assume that these securities are homogeneous and the only earning asset available to commercial banks.² Finally, in order to focus on the short run, we introduce the device of a finite interval (reminiscent of the Hicksian "week") during which portfolio plans are executed inflexibly on the basis of decisions made

² Alternatively, the optimum composition of portfolio assets is treated as a problem conceptually distinct from that of optimum portfolio size. Such an approach is pursued in J.L. Pierce, "The Monetary Mechanism: Some Partial Relationships", American Economic Review (Papers and Proceedings), LIV, 1964, pp. 523-531.
at the beginning of the period. Our theory concerns the "week-to-week" responses of individual banks, not long-run positions of portfolio rest. It is, therefore, a theory of flow equilibrium only.

At the beginning of each week, banks are assumed to face a specified need for new reserves ($\Delta R$), a given rate of interest on marketable securities ($r_s$), and a given discount rate ($r_d$). Each banker possesses only one decision variable: the total amount of borrowing ($B$) he will undertake over the upcoming period. Sales of securities ($- \Delta S$) plus borrowing must add up to $\Delta R$, and negative borrowing (at the discount rate) is not allowed. Finally, we assume that borrowing from previous periods expires automatically.

Next, we postulate that bank portfolio managers determine each period's borrowing (and consequently adjust their security holdings) so as to strike a balance between the cost of raising whatever reserves they require ($C$) and the disutility which arises from increased debt to the Federal Reserve.

Using the symbols we have just defined, we can write the cost of acquiring a given amount of reserves as:

$$C = C(B, \Delta S) = r_d B - r_s \Delta S.$$  \hspace{1cm} (1a)

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\(^3\)This formulation is, of course, strictly an approximation. We neglect both (1) the possibility of adjusting reserves through the federal-funds market (though conceivably we could interpret such adjustments either as securities' or borrowing transactions) and (2) the fundamental bank decisions which generate the $\Delta R$'s we take as given. It should be noted that portfolio adjustments may vary with the precise source of reserve needs (deposit withdrawals, loan requests, etc.). We return to this possibility in Sect. II.
Letting \( k \) stand for the algebraic difference between \( r_s \) and \( r_d \) and making use of the (assumed) identity \( \Delta R = B - \Delta S \), we can rewrite (1a) as follows:

\[
C(B, \Delta S) = r_s \Delta R - k B.
\]

(1b)

According to our postulate, given the values of the exogenous variables \( r_s, r_d, \) and \( \Delta R \), portfolio managers act as if to maximize an objective function of the form:

\[
U = U(C, B); \quad \frac{\partial U}{\partial C}, \frac{\partial U}{\partial B} < 0,
\]

(2)

subject to the side conditions (1b) and

\[
-\Delta S \leq S_0 \quad \text{(3a)} \\
B \geq 0. \quad \text{(3b)}
\]

Conditions (3a) and (3b) state that the banks cannot sell more securities than they own and that borrowing cannot be negative. If constraint (3a) is assumed to be nonbinding for periods as short as the basic accounting period and if \( r_s \) and \( k \) are given, then from (1b) we have

\[
dC = -kdB. \quad \text{(4)}
\]

Taking the first and second differentials of (2) yields

\[
dU = U_1 dC + U_2 dB, \quad \text{(5)}
\]

\[
d^2U = U_{11} (dC)^2 + U_{21} dB dC + U_{12} dC dB + U_{22} (dB)^2. \quad \text{(6)}
\]

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4 An equivalent and more natural formulation would view portfolio managers as minimizing a disutility function, i.e., one with positive first partial derivatives. The formulation in the text follows Polakoff, op. cit.

5 Should we introduce the federal-funds market for individual banks, only one of these restrictions could remain in force.
Substituting (4) into these expressions and taking $U_{12}$ and $U_{21}$ equal yields

$$\begin{align*}
dU &= (U_2 - k U_1) dB , \\
d^2U &= (U_{11} k^2 - 2 U_{12} k + U_{22})(dB)^2.
\end{align*}$$

For an interior maximum, we must have $dU = 0$ and $d^2U < 0$. These first- and second-order conditions can be written as

$$\begin{align*}
\frac{U_2}{U_1} &= k , \\
U_{11} k^2 - 2 U_{12} k + U_{22} &< 0 .
\end{align*}$$

In order to calculate an optimal $B (B^*)$ for given $k$ and $\Delta R$, one has only to solve (9) and (1b) simultaneously. Since (1b) involves the given initial reserve need, the optimal $B$ (and its complement the optimal $\Delta S$) clearly depend on $\Delta R$ as well as on $r_s$ and $k$.  

Let $B^*(k, r_s, \Delta R)$ denote the optimal level of borrowing expressed as a function of $k$, $r_s$, and $\Delta R$. If our theory is to be testable, it must place restrictions on the partial derivatives of this function, particularly on the signs of $\frac{\partial B^*}{\partial k}$, $\frac{\partial B^*}{\partial r_s}$, and $\frac{\partial B^*}{\partial (\Delta R)}$.

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6 The possibility of a corner or boundary maximum is, of course, very real here. For example (9) and (10) might imply a $B < 0$. In this case, so long as (10) holds, $B = 0$ is optimal. Given the assumptions we have introduced thus far, it is also possible that (10) may never be satisfied. In that case, banks would borrow indefinitely large amounts to finance security purchases. This pathological case is ruled out by the introduction of Federal Reserve surveillance below. On the other hand, we cannot rule out the possibility that, on occasion, the solution to (9) and (10) would be such that $B > \Delta R$.

7 In practice, except on the rare occasions when the discount rate is changed, $r_s$ and $k$ move together so that their separate effects cannot usually be distinguished. See below.
As in the Slutsky Equation of ordinary demand theory, these restrictions are found by taking the total differential of the first-order condition (9) in the vicinity of the equilibrium. First, we rewrite (9) as

$$ U_2(C^0, B^0) = k U_1(C^0, B^0) \quad (11) $$

Equating the total differentials of both sides and suppressing the superscripts, we get.

$$ U_{21} dC + U_{22} dB = k U_{11} dC + k U_{12} dB + U_1 dk \quad (12) $$

Next, from (1a) we find the total differential, dC:

$$ dC = - k dB - B dk + r_s d(\Delta R) + \Delta R dr_s $$

Substituting this into (12), we obtain

$$ U_{21}[- k dB - B dk + r_s d(\Delta R) + \Delta R dr_s] + U_{22} dB $$

$$ = k U_{11}[- k dB - B dk + r_s d(\Delta R) + \Delta R dr_s] + k U_{12} dB + U_1 dk $$

Combining terms in the various differentials and making use of the equality between $U_{12}$ and $U_{21}$, this may be written as

$$ dB[U_{22} - 2 k U_{12} + k^2 U_{11}] = dk[(U_{12} - k U_{11}) B + U_1] $$

$$ + d(\Delta R) r_s[k U_{11} - U_{12}] + dr_s(\Delta R)[k U_{11} - U_{12}] \quad (13) $$

This expression allows us to evaluate each of the important partial derivatives: $\frac{\partial B}{\partial (\Delta R)}$, $\frac{\partial B}{\partial r_s}$, and $\frac{\partial B}{\partial k}$. All of these derivatives have as their denominator the coefficient of dB in (13)\footnote{For example, $\frac{\partial B}{\partial (\Delta R)}$ is obtained from (13) by setting $dr_s$ and $dk$ equal to zero and solving.}, which, as the left-hand side of
the second-order maximum condition (10), must be negative.

First, consider \( \partial B / \partial (\Delta R) \). For \( r_s \) positive and greater than \( r_d' \) we would expect on a priori grounds that borrowing would rise with total reserve need. This requires \( k \frac{U_{11} - U_{12}}{U_{11}} < 0 \). This is, of course, an assumption and need not hold empirically over the entire range of \( \Delta R \). On the other hand, the very notion of banks' borrowing to meet reserve needs suggests that the derivative in question will normally be positive.

Before attempting to assess the other derivatives we want to emphasize that some care is needed in their interpretation. The problem is that the interest-rate variables, \( k \) and \( r_s \), which appear on the right side of (13) are not independent. Hence, in solving (13) mechanically for the partial derivative of \( B \) with respect to \( r_s \), we impose the condition that \( k \) be constant. To indicate this, we write this derivative as \( \frac{\partial B}{\partial r_s} \bigg|_{k=k} \). This means that we are assessing the effects of simultaneous and equal changes in \( r_s \) and \( r_d' \). Similarly, the partial derivative of \( B \) with respect to \( k \) is taken with \( r_s \) constant. This states the effects of a ceteris paribus change in \( r_d' \), and we write it as \( \frac{\partial B}{\partial k} \bigg|_{r_s=r_s} \).

As far as empirical work is concerned, the \( k \) observed ordinarily involves a simultaneous change in \( r_s \) and \( k \). Hence, the relevant derivative is

\[
\frac{\partial B}{\partial k} = \left( \frac{\partial B}{\partial k} \right)_{r_s=r_s} + \left( \frac{\partial B}{\partial r_s} \right)_{k=k}.
\]

\[9\] The intrepid reader may find it instructive to derive an expression for the effect of simultaneous but unequal changes in \( r_s \) and \( r_d' \).
Turning to the interest-rate derivatives, we see that, for positive \( \Delta R \), the sign of \( \frac{\partial B}{\partial r_s} \bigg|_{k=k} \) is the same as that of \( \frac{\partial B}{\partial (\Delta R)} \). Thus, in the "normal" case equal increments in \( r_s \) and \( r_d \) will also lead to an increase in borrowing: \( \frac{\partial B}{\partial r_s} \bigg|_{k=k} > 0 \). This brings us to \( \frac{\partial B}{\partial k} \bigg|_{r_s=r_s} \), the sign of which depends on the sign of \( B[U_{12} - kU_{11}] + U_1 \). Because the bracketed term is the negative of \( [kU_{11} - U_{12}] \), its sign has already been fixed. It will normally be positive. Since \( U_1 \) is negative, the sign of \( \frac{\partial B}{\partial k} \bigg|_{r_s=r_s} \) cannot in this case be unambiguously predicted. However, several definite statements can be made. For \( k \) and \( \Delta R \) such that the optimal \( B \) is in the vicinity of zero, the negative \( U_1 \) term predominates and the derivative must itself be positive.

At \( B = 0 \), the pure profit theory of borrowing applies, i.e., a decrease in the discount rate (\( r_s \) constant) with no change in reserve "need" will lead to an increase in borrowing. For \( B \) sufficiently large, \( \frac{\partial B}{\partial k} \bigg|_{r_s=r_s} \) changes sign; need and/or reluctance become the dominant factors.

As we have indicated, the relevant partial derivative, \( \frac{\partial B}{\partial k} \), is the sum of the previous two derivatives. Its sign is that of

\[
(B - \Delta R)(k U_{11} - U_{12}) - U_1.
\]

This sign varies with \( B \) and, in what we have called the normal case, will be negative only when \( B \) exceeds \( \Delta R \). This means that borrowing would be supporting net additions to security holdings. It is precisely this kind of borrowing that the Federal Reserve has unambiguously condemned. But before proceeding to complicate matters with Federal Reserve surveillance,
we should observe that our theory of the sign of $\frac{\partial B}{\partial k}$, while similar, is
different from that of Polakoff. Whereas Polakoff's work ignores import-
ant ceteris-paribus conditions, we allow specifically for variability in the
weekly reserve need and provide a more extended analysis of the role of
interest rates.  

B. Introducing Federal Reserve Surveillance

In recent years, Federal Reserve officials have been especially
careful to emphasize that member-bank borrowing is a privilege of system
membership and by no means a right. 11 The practical implication of this
distinction is that Federal Reserve authorities reserve the right to challenge
member-bank use of the borrowing privilege. In a series of pronouncement-
ments dating from 1951-52, 12 System officials have made it abundantly
clear that extensive member-bank recourse to the discount window can

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10 With a few emendations, Polakoff's formulation (op. cit., pp. 6-8) would
emerge as a special case of our own, viz. where $\Delta R$, member-bank need for
reserves, is constant over time. While Hodgman speaks of more ultimate
variables (the demand for bank loans and Federal Reserve open-market opera-
tions), this appears to be the force of the objections in Hodgman, op. cit.

11 See, for example, the account of current discount policy in G. W. McKinney,
Jr., The Federal Reserve Discount Window: Administration in the Fifth
District, Rutgers University Press, New Brunswick: 1960, Chapter VII; or
the famous exchange between Whittlesey and Roosa, "Credit Policy at the
Discount Window", Quarterly Journal of Economics, 1958, pp. 207-216 and
1959, pp. 33-38.

12 Since borrowing could be used to expand a bank's taxable base, banks
seeking relief from the excess-profits tax then in force caused borrowings
to attain record levels. We might observe that, because of this specialized
influence, we begin our empirical analysis in mid-1953.
easily represent an abuse of this privilege, and that individual banks whose borrowings are deemed improper or excessive will be subject to certain ill-defined sanctions.

So far, we have offered no justification for having banks regard borrowing as a source of disutility, i.e., for making borrowing a specific argument of the typical bank's objective function. We have in mind, however, the reputed tradition against borrowing as a less than fully respectable way of raising commercial-bank reserves.

Of course, the attitudes underlying this tradition are subject to change. In particular, they can be tempered or reinforced by Federal Reserve pronouncements and discount policy. Witness, for example, the Federal Reserve Board's 1955 revision of Regulation A. By explicitly declaring that borrowing for certain purposes was improper, the new regulation supported any preexisting (and in some cases created) member-bank bias against borrowing.

Besides moral suasion associated with announcements and explanations of basic changes in Regulation A, the Federal Reserve exercises a tangible influence on the nonrate costs of member-bank borrowing. It does this through its continuing surveillance of the use being made of the discount window. While the simple fact of surveillance tends to intensify bankers' reluctance to borrow, it also makes transactions more tedious, thereby increasing the real costs of borrowing reserves from the Federal
Reserve. For example, banks which have been borrowing regularly will be subject to Federal-Reserve demands for additional records or officer interviews. Because of these additional costs, even if the transactions costs of borrowing were normally identical for all banks (as they are undoubtedly not) the rate spread observed in the market place \((k = r_s - r_d)\) is not the relevant opportunity cost of borrowing for individual banks.

We can allow for this by introducing an implicit discount rate, \(r_d'\), which exceeds \(r_d\) by the amount of the imputed transactions cost, \(c\).

On the assumption that Federal-Reserve harassment varies with the use made of its discount windows, \(c = r_d' - r_d = \Phi(B)\), where \(\Phi'(B) > 0\).\(^{13}\)

From this, it follows that the relevant least-cost spread or differential opportunity cost may be written \(k' = r_s - r_d' = k - \Phi(B)\).

Substituting the function \(k'\) for \(k\) in (1b) changes the solution of the bank's portfolio problem. Instead of (9), the necessary condition for optimal \(B\) becomes

\[
\frac{U_2}{U_1} = k - \Phi(B) - B \Phi'(B) = k' - B \Phi'(B). \tag{14}
\]

From (14) we can derive expressions analogous to those presented in Section A. for \(\partial B / \partial k\), \(\partial B / \partial r_s\) and \(\partial B / \partial (\Delta R)\). However, since these rest on an oversimplified static view of surveillance they would not add significantly to the discussion.

\(^{13}\) Once again, this is merely a static first approximation. In the real world, the Federal Reserve appears much more concerned about the level of average borrowings over time than about borrowings in any one week. This suggests that the imputed transactions cost should be regarded as a function of average borrowings in the recent past. Such a formulation, which introduces an obvious time dependence between borrowing decisions made in adjacent weeks, is treated in Section II.
One difference in the analysis is the possibility of variations in the surveillance function $\Phi(B)$. For example, the effect of $\Phi''$ (the rate at which additional borrowing increases costs imposed by surveillance) is unambiguous. Because $C(B, \Delta S)$ is independent of $\Phi''$, for $B > 0$, it is easy to show that $\frac{\partial B}{\partial k}$ and $\frac{\partial B}{\partial(\Delta R)}$ are each reduced by increases in $\Phi''$.

II. DISTRIBUTED-LAG ADJUSTMENT

Thus far, we have depicted the member-bank borrowing decision as operating in a wholly static frame. We call out $\Delta R$, $r_s$, and $k$, and our theory grinds out an optimal level of borrowings, $B^0$. In the real world, of course, bank portfolio adjustments do not take place timelessly. This is as true for the individual banks as for the banking system as a whole, and (as indicated in footnote 13) is made all the truer by the fact of Federal Reserve surveillance. Borrowing requests of banks which have been long or frequently in debt have a more difficult course to run.

In this section, we wish to identify and to explain two additional sources of dynamic complications:

1. implicit and explicit transaction costs which rule out statically complete balance-sheet adjustment within short periods of time;

2. uncertainty regarding the permanence of recent changes in $R$. 
The first of these factors supplemented by the effects of Federal Reserve surveillance will be seen to imply that a bank's demand for current borrowing will depend, in part, on the pattern of its borrowing in the recent past. The second factor supports the inclusion of current and lagged flows of unborrowed reserves \((\Delta U_t, \Delta U_{t-1}, \ldots)\) in the borrowing function. Taken together with the analysis of Section I, these considerations lead to an expression for optimal borrowing of the following type:

\[
B_t^0 = B^0(k_t, B_{t-1}, B_{t-2}, \ldots; \Delta U_t, \Delta U_{t-1}, \ldots)
\]  

(15)

In recent empirical work, functions like (15) which incorporate distributed lags have become more and more widely applied. Of particular interest, Meigs has used such a function in order to account for the closely related variable, free reserves.

A. Transactions Costs, Surveillance, and Dynamic Adjustments

Transactions costs can make it uneconomic to adjust to static optimality within a single week. When a reserve deficiency is only temporary, the turnaround cost of disinvesting and later reacquiring a similar security may be quite high relative to the cost of borrowing. Even when reserve deficiencies are permanent, borrowing may for a number of weeks represent the preferred source of finance.

To see this most clearly, we have only to recognize that bank "securities" differ in at least two important respects: in their marketability
and in their availability for immediate sale. Decreases in loans can be effected only as existing loans mature. Highly marketable securities may be tied up as pledges against public deposits. Also, on occasion, tax considerations (gain- and loss-year decisions\(^{14}\)) may make it wise to delay security sales until the next calendar year. All of this means that in the very short run a bank may prefer to borrow rather than to rid itself immediately of the securities it plans ultimately to sell.

These considerations are reinforced by a second factor. Legend has it that Federal Reserve authorities and bankers themselves attach less stigma to temporary than to longer-run borrowing.\(^{15}\) Hence, borrowing specifically undertaken to ease (but not to escape) the rigors of the statically-optimal adjustment should be associated with relatively low psychic cost and surveillance-induced transactions expense.

The foregoing analysis suggests why banks are apt to borrow more in the very short run than they would in a purely static model. It also suggests that following an exogenous disturbance, banks effect desired changes in their security portfolios through a series of partial adjustments. Since in our model there are only two alternative means of raising reserves, this justifies the presence of lagged Bs in (15). Surveillance-induced transactions costs that vary directly with past borrowing activity provide still another

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\(^{15}\) Federal Reserve surveillance aims, in fact, as assuring that in the long run B\(^{0}\) equals zero.
justification. Such costs mean that, while the maturation of past borrowing produces a continuing need for funds, these borrowings become more expensive to service the longer they remain in force.

Taken together, these points lead us to expect that current borrowings will vary positively with the level of borrowing in previous weeks, with the influence of past borrowings falling off (and perhaps, because of surveillance costs, even becoming negative) as they recede into the more and more distant past.

B. Permanent Versus Temporary Reserve Deficiencies

The first thing we must recognize is that $\Delta R$, the weekly reserve need is an unobservable variable. In the final analysis, it depends on bank decisions regarding desired weekly expansions and on autonomous flows of unborrowed reserves. If we are to make our theory testable, we must find some way of giving this variable operational significance.

To this end, we assume that, as a matter of overall portfolio policy, each bank portfolio manager sets a constant target of weekly asset change. This assumption is, of course, an arbitrary one and in the nature of a strategic simplication rather than an assertion of fact. Now, let us suppose that bankers find it convenient to decompose reserve deficiencies into a permanent and a transitory component:
\[ \Delta R_t = \Delta R_t^P + \Delta R_t^T, \]  

(16)

where \( \Delta R_t^T \) is regarded as random. This distinction, pioneered and employed so fruitfully by Milton Friedman, is made operational by positing that permanent quantities are estimated as weighted averages of past observations, with the weights involved declining as variables recede into the more distant past.

By definition, each permanent \( \Delta R \) would emerge as the difference between \( A \), the change in required reserves implied by the desired change in assets, and the banker's estimate of the permanent flow of unborrowed reserves:

\[ \Delta R_t^P = A - \Delta U_t^P \]  

(17)

The concept of a permanent flow of unborrowed reserves is an important one. It represents the variable whereby banks incorporate into their borrowing decisions long-run developments in their competitive positions and in Federal Reserve open-market policy. Following the approach outlined above, we write banker estimates of \( \Delta U_t^P \) as:

\[ \Delta U_t^P = \sum_{i=0}^{h} \omega_{t-i} \Delta U_{t-i}, \]  

(18)

where the horizon \( h \) and the weights \( \omega_{t-i} \) will have to be determined empirically. Substituting (18) into (17) and the result into (16), we get

\[ \Delta R_t = A - \sum_{i=0}^{h} \omega_{t-i} \Delta U_{t-i} + \Delta R_t^T. \]  

(19)
Replacing \( \Delta R_t \) by this expression and inserting lagged values of \( B \) into the static \( B^0 \)-function produces a stochastic version of (15)

\[
B_t^0 = B^0(k; B_{t-1}, B_{t-2}, \ldots; \sum_{i=0}^{h} \omega_{t-i} \Delta U_{t-i}) + v_t
\]  

(20)

In this equation the random-error term \( v_t \) represents the nonpredictable components of \( \Delta R_t \) and specification error associated with the assumption of an unchanging target of asset expansion.

The versions of (II.6) we estimate in Section IV will include both linear and higher order terms in \( k_t \) (to test the Polakoff hypothesis that \( \frac{\partial B}{\partial k} \) declines with \( k \)) and will be linear in the \( B_{t-i} \) and in \( \sum_{i=0}^{h} \omega_{t-i} \Delta U_{t-i} \).

As a representative example, consider the equation

\[
B_t = a_0 + a_1 k_t + a_2 k_t^2 + a_3 B_{t-1} + a_4 B_{t-2} + a_5 B_{t-3} + a_6 \Delta U_t + a_7 \Delta U_{t-1} + a_8 \Delta U_{t-2} + v_t^f.
\]  

(21)

According to Polakoff \( a_2 \) should be less than zero, while the theory we have sketched implies the following pattern of signs and magnitudes:

\[
a_1 > 0; \\
a_3 > 0, a_3 > a_4 > a_5; \\
a_6 < 0, a_6 < a_7 < a_8.
\]
III. AGGREGATION PROBLEMS

Since the objective functions (2) are likely to vary from bank to bank, a proper test of the theory of Sections I and II would make use of data recording the responses of individual banks. Unfortunately, such data are not available. The least aggregate of available figures portray totals for each of four broad classes of member banks.

While these groupings are far from homogeneous, within each category the pattern of responses ought to be more homogeneous than across the universe of all member banks. In any case, this is a testable hypothesis we can and will subsequently examine.

Variation within classes is something else again. All we can do is to point to obvious differences in members' basic situations (in portfolio size, location, service and deposit mix, competitive pressures, etc.). These differences seem so considerable as to make it quite likely that members' responses would show important differences in magnitude and that further disaggregation would increase our ability to predict borrowing behavior.

For each class, besides differences in individual coefficients of borrowing response, the microeconomic variables to which individual members respond may not be well represented by observed k's or aggregate ΔU's and lagged borrowings. Consider k. Given the facts of Federal
Reserve surveillance and of differential transactions costs in marketable securities (e.g., because of physical distance from securities markets or informational deficiencies which must be overcome), the k's to which different banks respond are not really the same. As a consequence, observed k's must be regarded as rough "average" figures and regression coefficients interpreted accordingly.

Moreover, with no provisions for federal funds, the weekly distribution of maturing borrowings and unborrowed-reserve flows become particularly important. A bank can use new reserves to repay borrowings only to the extent it was previously in debt. Hence, starting from a zero-borrowing equilibrium, a zero net change in the unborrowed reserves of any bank class (k and $r_s$ unchanged) does not rule out the possibility that various members of the class find it advantageous to borrow.

As the borrowing banks repay their debt, there is set in motion a systemic multiple contraction of deposits originating in the net security purchases undertaken by those banks that had originally gained reserves. These multiplier chains are apt to develop at least partially through different banks and proceed through the different classes at somewhat different speeds. As a result, a zero net change in the reserves of one class can disturb the equilibrium of all classes. In general, such distribution effects ought to be more important, the greater the net flow of deposits within any class over its particular reserve-computation period and the greater interaction among members of different classes.
Since deposit flows and the network of interaction are apt to vary seasonally, distribution effects may emerge as a particularly important determinant of seasonal patterns in borrowing.\(^{16}\) Also, because of the non-negativity constraint on borrowing by individual banks, security markets are not immediately called upon to reconcile selling pressure to the full extent of emerging demands. Hence, distribution effects may lead both to more borrowing over time and to somewhat slower overall adjustments than would occur in their absence.

On the other hand, it is easy to make too much of these matters. Except as a possible further explanation for seasonal and inertial patterns, in this study we do not specifically invoke, nor do we isolate, distribution effects. They remain important mainly as a problem for further research (how to take account of interaction between the borrowing patterns of individual banks and classes of banks) and as a warning that the microeconomic interpretation of the aggregate coefficients we estimate is a somewhat slippery task.

\(^{16}\) This suggests that in empirical work it will be advisable to correct for seasonal variation.
IV. EMPIRICAL RESULTS

In this section we provide empirical support for the basic model developed above. The evidence presented is obtained from estimation of linear variants of equation (20). In examining alternative specifications we concentrate on questions of the appropriate lag patterns and the form of the cost variable, k.

We use weekly data on reserves and borrowings for each of four member-bank categories and the discount and Treasury bill rates. As sales of short-term government securities provide the most likely alternative to borrowing as a means of raising funds, the Treasury bill rate seems a reasonable choice for \( r_s \).\(^{17}\) The data were taken from various issues of the Federal Reserve Bulletin and cover the period from July 1953 to December 1963 - a total of 542 observations.\(^{18}\) Dollar variables are measured in millions while the interest variables are measured in percent. The starting date was chosen, as indicated above, largely to eliminate the impact of the excess-profits tax which coincided with record-level borrowings in the second half of 1952 and the first half of 1953.

\(^{17}\) In keeping with our discussion above we did not enter the bill rate as a separate variable but utilized the differential, k. There is precedent for both the choice of \( r_s \) and the use of a differential, see Meigs, op. cit.

\(^{18}\) The data start at the beginning of July but as we make use of lagged variables several observations are lost.
A. *Seasonally-unadjusted Results*

Our initial attempts at estimation made use of the data in seasonally unadjusted form and a sample equation for each bank class appears in Table I. All of the variables obtain signs which are in accord with our expectation and virtually all achieve statistical significance at the 5 per cent level. The two exceptions to this are the cost variable for the Chicago banks and lagged unborrowed reserves for Other Reserve City banks. On the whole, however, the results of Table I provide clear support for the importance of current and lagged unborrowed reserves, lagged borrowings and the cost variable.

We have already suggested that distribution effects make it likely that the raw data will exhibit seasonal variation. In addition, there are other (more standard) reasons for suspecting that seasonal variation may be a problem in this context. In view of this, we shall not analyze the results of Table I in any detail but rather turn directly to the issue of seasonal adjustment.

B. *Seasonal Adjustment*

As is becoming increasingly well known, procedures designed to remove seasonal variation from a time series can also alter the non-seasonal characteristics of the series. More explicitly, recent work using the techniques of spectral and cross-spectral analysis has shown
<table>
<thead>
<tr>
<th>Class</th>
<th>Const.</th>
<th>B&lt;sub&gt;-1&lt;/sub&gt;</th>
<th>B&lt;sub&gt;-2&lt;/sub&gt;</th>
<th>ΔU</th>
<th>ΔU&lt;sub&gt;-1&lt;/sub&gt;</th>
<th>ΔU&lt;sub&gt;-2&lt;/sub&gt;</th>
<th>(r&lt;sub&gt;s&lt;/sub&gt; - r&lt;sub&gt;d&lt;/sub&gt;)</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>New York</td>
<td>22.23</td>
<td>0.684</td>
<td>0.093</td>
<td>-0.369</td>
<td>-0.059</td>
<td>-</td>
<td>25.33</td>
<td>0.722</td>
</tr>
<tr>
<td></td>
<td>(3.78)</td>
<td>(0.043)</td>
<td>(0.043)</td>
<td>(0.020)</td>
<td>(0.026)</td>
<td>-</td>
<td>(5.92)</td>
<td></td>
</tr>
<tr>
<td>Chicago</td>
<td>2.40</td>
<td>0.775</td>
<td>0.183</td>
<td>-0.775</td>
<td>-0.143</td>
<td>-</td>
<td>2.40</td>
<td>0.939</td>
</tr>
<tr>
<td></td>
<td>(1.10)</td>
<td>(0.042)</td>
<td>(0.042)</td>
<td>(0.017)</td>
<td>(0.037)</td>
<td>-</td>
<td>(1.77)</td>
<td></td>
</tr>
<tr>
<td>Other Reserve City</td>
<td>16.53</td>
<td>0.798</td>
<td>0.152</td>
<td>-0.440</td>
<td>-0.006</td>
<td>-</td>
<td>18.93</td>
<td>0.936</td>
</tr>
<tr>
<td></td>
<td>(4.28)</td>
<td>(0.043)</td>
<td>(0.042)</td>
<td>(0.019)</td>
<td>(0.027)</td>
<td>-</td>
<td>(5.66)</td>
<td></td>
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<td>Country</td>
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<td>0.502</td>
<td>0.430</td>
<td>-0.125</td>
<td>-0.099</td>
<td>-0.097</td>
<td>10.62</td>
<td>0.886</td>
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<tr>
<td></td>
<td>(2.31)</td>
<td>(0.035)</td>
<td>(0.035)</td>
<td>(0.008)</td>
<td>(0.010)</td>
<td>(0.008)</td>
<td>(2.70)</td>
<td></td>
</tr>
</tbody>
</table>

* The numbers in parentheses are the standard errors of the individual coefficients. The unborrowed reserve variables are constructed for each class separately.
that procedures aimed at eliminating variation at seasonal frequencies may change the series at other frequencies as well.\(^{19}\) These difficulties plus the fact we are working with weekly data led us to choose a rather simple procedure which nevertheless appeared to accomplish our objective. Starting with a series \(X(t)\) we took deviations from a one-year-centered moving average, i.e., computed

\[
S(t) = X(t) - \frac{1}{26} \sum_{h=-26}^{26} X(t + h),
\]

which can be viewed as a series of seasonals. In order to obtain one seasonal factor per week we took the mean value of the series for each week separately. We then subtracted this result from the original series to obtain the seasonally adjusted series. For example, we found the mean of all the \(S(t)\) in our sample corresponding to the first week of the year and subtracted this same value from every \(X(t)\) which also was a first week. Implicit in this is an assumption of stationarity of the seasonal but given the nature of the various series, this does not seem unreasonable.

In order to see if we had effectively removed the seasonal we computed spectra and cross-spectra of the adjusted and unadjusted series. Briefly, those results indicated that we had removed a significant part of the seasonal and had not altered the series in any important way. We now turn to results using the seasonally-adjusted data.

C. Seasonally-adjusted Results

Table II presents the basic results. Contrasting the comparable specifications in Tables I and II (the relevant equations are starred in Table II) reveals that several systematic biases result from the use of the unadjusted data. In particular, the adjusted data produce uniformly lower coefficients for one-period lagged borrowing and uniformly higher coefficients for two-period lagged borrowing. Thus the dynamic implications of the two sets of results are different. There are also systematic differences between the results with respect to the cost differentials (lower for adjusted data) and unborrowed reserves (higher for adjusted data). As expected, the coefficients of determination for comparable specifications are uniformly higher for the adjusted equations. It would seem that seasonal adjustment is an important component of an analysis of weekly borrowing data. The remainder of the paper concentrates on the adjusted results.

Despite their quantitative differences, both the seasonally-adjusted and unadjusted results are qualitatively consistent with the basic framework of the paper. As indicated above, however, the evidence presented in Table II provides even stronger support for the importance of distributed-lag responses. Let us briefly examine the results in Table II, simultaneously pointing up some of the more interesting class differences.

---

20 The use of the term "bias" rests on an assumption about the proper form of specification.
<table>
<thead>
<tr>
<th>Class</th>
<th>Const.</th>
<th>B_1</th>
<th>B_2</th>
<th>B_3</th>
<th>ΔU</th>
<th>ΔU -1</th>
<th>ΔU -2</th>
<th>(r_s - r_d)</th>
<th>(r_s - r_d)^2</th>
<th>(r_s - r_d)^3</th>
<th>R^2</th>
<th>s.e.</th>
</tr>
</thead>
<tbody>
<tr>
<td>* New York</td>
<td>15.49</td>
<td>.673</td>
<td>.156</td>
<td>-</td>
<td>-.409</td>
<td>-.100</td>
<td>-</td>
<td>16.12</td>
<td>(5.01)</td>
<td>-</td>
<td>.753</td>
<td>41.12</td>
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<td></td>
<td>(3.27)</td>
<td>(.043)</td>
<td>(.043)</td>
<td></td>
<td>(.021)</td>
<td>(.027)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>13.27</td>
<td>.662</td>
<td>.096</td>
<td>.095</td>
<td>-.400</td>
<td>-.096</td>
<td>-</td>
<td>13.90</td>
<td>(5.04)</td>
<td>-</td>
<td>.757</td>
<td>40.85</td>
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<td>(.042)</td>
<td>(.047)</td>
<td>(.033)</td>
<td>(.021)</td>
<td>(.027)</td>
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<td>-</td>
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<td>-.103</td>
<td>-</td>
<td>22.23</td>
<td>(7.98)</td>
<td>-24.20</td>
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<td>(.042)</td>
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<td>(.021)</td>
<td>(.027)</td>
<td></td>
<td></td>
<td>(16.99)</td>
<td>(-26.85)</td>
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<td></td>
</tr>
<tr>
<td>* Chicago</td>
<td>1.47</td>
<td>.712</td>
<td>.259</td>
<td>-</td>
<td>-.799</td>
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<td>-</td>
<td>1.67</td>
<td>(1.49)</td>
<td>-</td>
<td>.951</td>
<td>12.78</td>
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<tr>
<td></td>
<td>(.97)</td>
<td>(.042)</td>
<td>(.042)</td>
<td></td>
<td>(.016)</td>
<td>(.037)</td>
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<td>.262</td>
<td>-</td>
<td>-.797</td>
<td>-.213</td>
<td>-</td>
<td>3.24</td>
<td>(2.39)</td>
<td>-9.08</td>
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<td>(.041)</td>
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<td>(.016)</td>
<td>(.037)</td>
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<td></td>
<td>(5.27)</td>
<td>(-9.39)</td>
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</tr>
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<td>(.041)</td>
<td></td>
<td>(.020)</td>
<td>(.029)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>* Country</td>
<td>9.93</td>
<td>.457</td>
<td>.477</td>
<td>-</td>
<td>-.076</td>
<td>-.024</td>
<td>-</td>
<td>10.64</td>
<td>(2.68)</td>
<td>-</td>
<td>.882</td>
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<tr>
<td></td>
<td>(2.35)</td>
<td>(.038)</td>
<td>(.038)</td>
<td></td>
<td>(.011)</td>
<td>(.011)</td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>9.48</td>
<td>.485</td>
<td>.455</td>
<td>-</td>
<td>-.086</td>
<td>-.093</td>
<td>-</td>
<td>9.26</td>
<td>(2.44)</td>
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<tr>
<td></td>
<td>(2.14)</td>
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<td>(.034)</td>
<td></td>
<td>(.010)</td>
<td>(.012)</td>
<td></td>
<td></td>
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<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>10.60</td>
<td>.452</td>
<td>.475</td>
<td>-</td>
<td>-.075</td>
<td>-.023</td>
<td>-</td>
<td>17.58</td>
<td>(4.43)</td>
<td>9.46</td>
<td>.883</td>
<td>22.59</td>
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<td>(2.37)</td>
<td>(.038)</td>
<td>(.038)</td>
<td></td>
<td>(.011)</td>
<td>(.011)</td>
<td></td>
<td></td>
<td>(4.81)</td>
<td></td>
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</tr>
<tr>
<td>* Total</td>
<td>33.55</td>
<td>.631</td>
<td>.315</td>
<td>-</td>
<td>-.246</td>
<td>-.120</td>
<td>-</td>
<td>35.92</td>
<td>(11.81)</td>
<td>-</td>
<td>.921</td>
<td>94.03</td>
</tr>
<tr>
<td></td>
<td>(9.32)</td>
<td>(.041)</td>
<td>(.040)</td>
<td></td>
<td>(.022)</td>
<td>(.025)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>32.21</td>
<td>.604</td>
<td>.254</td>
<td>.092</td>
<td>-.237</td>
<td>-.118</td>
<td>-</td>
<td>36.43</td>
<td>(11.76)</td>
<td>-</td>
<td>.922</td>
<td>93.62</td>
</tr>
<tr>
<td></td>
<td>(9.30)</td>
<td>(.042)</td>
<td>(.048)</td>
<td>(.038)</td>
<td>(.023)</td>
<td>(.024)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
For all classes and for the total, borrowing lagged one and two periods are important elements of the equations but the pattern of coefficients is different across classes. For New York banks the drop in magnitude between the coefficients of $B_{-1}$ and $B_{-2}$ is greatest while at the other extreme country banks exhibit virtually no decline in these coefficients. In fact, in two of the country-bank equations the coefficient of $B_{-2}$ is larger than that of $B_{-1}$. This reflects the slower speed of adjustment of country bank portfolios - a finding which is consistent with other studies.\footnote{See S. M. Goldfeld, \textit{Commercial Bank Behavior and Economic Activity}, North-Holland Publishing Co., 1966. One other result not reported in Table II should be noted. When $B_{-4}$ was added to the various equations it typically yielded a positive and significant coefficient. However, $B_{-3}$ was rarely significant in these cases. In addition, it was sometimes negative. While this is not bad in itself - in fact, it is what the surveillance argument might suggest - the presence of multicollinearity introduced by the use of four lagged borrowing variables led us to discount these results.}

As for changes in unborrowed reserves, all bank classes yield significant coefficients for both one and two period lags although the pattern of coefficients is quite diverse. Once again, the effect of the more distant past tends to be smaller than the more recent past. As with the lagged-borrowing variables, the one exception to this is the country-bank class, which, it might be noted, was the only class to yield a significant coefficient for $\Delta U_{-2}$.

There is similar variation across classes in the response of banks to interest rate differentials. The short-run interest rate elasticities with respect to the bill rate (evaluated at the means and calculated from the starred equations in Table II) are as follows: \footnote{The long-run elasticities range from 2.8 to 3.9. This is consistent with work using quarterly data. See Goldfeld, \textit{op. cit.}} New York - .56, Chicago -
.08, Other Reserve City - .15, Country - .21, Total - .21. New York is the most responsive and Chicago, rather unexpectedly, the least.

On the whole, therefore, while the basic framework of the paper accounts for member-bank borrowing in each bank class, it does so with differing quantitative implications and degrees of success (witness the variation in $R^2$). On a more formal level one can apply a covariance analysis to test the hypothesis that the same set of coefficients is appropriate to each bank class. Not surprisingly, when this was done this hypothesis was rejected at the one-per-cent level of significance.

Table II also casts light on one final issue. We earlier noted Polakoff's contention that the partial derivative of borrowing with respect to the relative cost variable, $k$, would become negative for high levels of $k$. This is clearly impossible if one utilizes only a linear term in $k$ in the regression equations. However, Polakoff suggests introducing a quadratic term in $k$ (which is expected to obtain a negative coefficient) so as to allow the marginal propensity to borrow with respect to cost to vary with the cost. For the period 1954-57, he has estimated

$$B = 806.2 + 210.9k - 1319.6k^2$$

which implies that $\partial B/\partial k$ is positive for $k$ less than .08 and negative for $k$

---

23 See M. E. Polakoff, "Federal Reserve Discount Policy and Its Critics", in Banking and Monetary Studies, ed. by D. Carson, Richard D. Irwin, Inc., 1963, pp. 190-212. No standard errors are provided for the coefficients although the author indicates the quadratic term is justified on the basis of an analysis of variance.
greater than .08. One of the defects of this equation is that it ignores the
affect of reserve needs on borrowing behavior. A comparison of simple
and partial correlation coefficients in Table III for the variables $\Delta U$ and $k$
indicates that this omission tends to produce more significant responses to
changes in $k$ than the data warrant. A fairer test would be to add a term
in $k^2$ to our regression equations. When this was done, the results did not
 corroborate Polakoff's findings. For three of the four bank classes (and
for the total) the coefficient of $k^2$ was positive and in fact significantly so
for the country class. The one negative coefficient was highly insignificant.
Only the result for the country banks is reproduced in Table II.

\begin{table}
\centering
\begin{tabular}{|l|cc|cc|}
\hline
 & $\Delta U$ & & $k$ & \\
 & Simple & Partial & Simple & Partial \\
Correlation & Correlation & & Correlation & Correlation \\
\hline
New York & -.24 & -.63 & .42 & .14 \\
Chicago & -.26 & -.90 & .33 & .05 \\
Other Reserve City & -.15 & -.73 & .52 & .11 \\
Country & -.08 & -.29 & .41 & .17 \\
TOTAL & -.07 & -.43 & .51 & .13 \\
\hline
\end{tabular}
\caption{Table III}
\end{table}

In order to further test the Polakoff hypothesis we added a term in $k^3$
(as well as $k^2$) to the various equations. As the simple correlation of $k^2$

\footnote{The partial correlation coefficients are from the starred equations in
Table II.}
and $k^3$ is -.94 we are treading on thin ice in attempting to ascribe much significance to these results. Nevertheless, it is instructive to note (see Table II) that for New York and Chicago both $k^2$ and $k^3$ obtain negative and nearly significant coefficients. The results for New York imply that for $k$ less than -.91 or greater than .30, the derivative $\partial B/\partial k$ will be negative. For Chicago, the corresponding estimates are -.80 and .15.\(^{25}\) There are a number of weeks in our sample for which the cost differential was outside these ranges and hence for which reluctance and/or surveillance appears to be relevant for at least part of the banking system.\(^{26}\) The absence of this effect in the other two bank classes may reflect genuine behavioral differences but might also result from statistical difficulties. In particular, the aggregation of numerous banks may obscure the desired result. On the other hand, the relatively small number of banks in the New York and Chicago classes may allow an aggregate equation to pick up the influence of surveillance more effectively.

V. CONCLUSION

The paper starts with a stylized model of bank-borrowing behavior which attempts to integrate profit, need, and surveillance considerations.

\(^{25}\) The fact that we get two estimates for each class reflects the use of a cubic equation in $k$.

\(^{26}\) It would, of course, be misleading to directly examine the changes in borrowing in these weeks as a check on this finding. This is precluded by the nature of a multiple regression equation.
The model is then embedded in a dynamic framework and the presence of lagged variables is justified. Following this, multiple regression equations are estimated for weekly data for four categories of member banks. The results clearly indicate the separate roles of changes in unborrowed reserves, lagged borrowings and cost considerations. Furthermore, they support other recent results which have found significant behavioral differences among various classes of banks.