ABSENCE OF MONEY ILLUSION: A SINE QUÆ NON

FOR NEUTRAL MONEY?

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Edward J. Kane† and Alvin K. Klevorick‡

General-equilibrium proofs of the neutrality of money rely on a set of
now-standard assumptions (the so-called neoclassical assumptions of monetary
theory) concerning price flexibility, expectations of future prices, distribution
effects, and money illusion.¹ Although the sufficiency of these assumptions
appears well-established, their necessity remains very much a topic for current
research.² This paper investigates one aspect of this topic. It shows that the
complete prohibition of money illusion, as in the conventional (or Patinkin)
formulation, is overly restrictive. An alternative formulation is developed in
which, in contrast to the Patinkin version, money illusion becomes a matter of
degree. This more general formulation leads directly to a weaker money-
illusion postulate, one which nevertheless suffices for neutrality.

For those who use the Patinkin model, this is surely a comforting result.

Because it is a razor's-edge case, the complete absence of money illusion is

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¹Money is neutral when the equilibrium values of the real system are inde-
dependent of the quantity of money in existence. The neoclassical assumptions are
summarized nicely in F. Modigliani, "The Monetary Mechanism and Its Inter-
action With Real Phenomena", Review of Economics and Statistics, Supplement

XXXI, April, 1964, pp. 147-148; F. Modigliani, op. cit.; and D. Patinkin,
Edition, especially Chapter XII where these and other assumptions are relaxed
one at a time.
an unlikely phenomenon indeed. So narrow a restriction makes one suspect instinctively any theory which depends critically upon it and makes one reluctant to take seriously the policy implications it may offer.

It turns out, however, that the Patinkin money-illusion postulate plays a very much smaller role in the theory of the real-balance effect than has been generally realized. So long as market participants show some (no matter how slight) degree of sensitivity to price-level change, the invalid dichotomy is destroyed. Moreover, something very much like a real-balance effect may be seen to operate with comparative-statics results similar to those obtained in models incorporating the strict Patinkin postulate.

The Concept of Money Illusion

In principle, money illusion arises whenever behavioral units overestimate the real value of their nominal wealth. Someone suffering from money illusion regards his stock of nominal assets as convertible into potentially more real goods and services ("commodities") than market prices actually allow. The essence of money illusion lies in such misperceptions of the price level \( P \): a weighted average of individual commodities' prices), the rate of exchange between nominal assets \( W \) and commodities.

Money illusion is thus really a matter of price-level illusion. The price level that appears in firm and consumer utility functions and to which they adjust their expenditure \( Q_1 \) is something other than what it ought realistically to be. Except as a special case, individuals respond not to real wealth per se, \( \frac{W}{P} \), but to their perceptions of this quantity, \( \frac{W}{P_1} \).
Intuitively, the difference between $P$ and $P_I$ could result from (1) an unreasonable weighting pattern (for example, zero weights for several relevant commodities, or even for all but one commodity), (2) lags in the processing of information on price changes, or (3) an habitual scaling-down of the effects of proportionate price movements. The latter case, which may occur either because of difficulties in collecting or interpreting reliable information or because of a habitually optimistic outlook, is the one on which we focus in this paper. We define the perceived value of nominal wealth as $\frac{W}{P_I}$, where $P_I = P^\alpha$, $0 \leq \alpha \leq 1$.

When $\alpha = 1$, the individual in question is free of money illusion in the Patinkin sense. Whenever $\alpha < 1$, money illusion is present in some degree, the degree being represented by the quantity $1 - \alpha$. In this way, the possibility $\alpha = 0$, which Patinkin has analyzed so extensively, emerges as the case of money illusion in the extreme.

The Neutrality of Money Under the Condition $0 < \alpha < 1$

In this section, we investigate how the substitution of perceived for real balances alters the analysis of money in general equilibrium. For this purpose, it suffices to concentrate upon the standard three-equation, outside-money model of Patinkin. In this flow-equilibrium model, the capital stock is ignored. Only

3 We recognize the possibility that the behavior of people, in particular the value of $\alpha$, may change over time. So as to avoid unduly complicating the analysis, however, we consider only the case in which $\alpha$ is constant. That is, we only consider what happens at a particular point in time or over a period of time short enough that $\alpha$ does not change during it.
three goods are recognized: commodities \((Y)\), bonds \((B)\), and money \((M)\). The first and third of these are supplied inelastically. Commodity output is fixed by an assumption of perpetual full employment and the money supply is determined exogenously by monetary authorities. The excess demand for each good is a function of three variables: commodity output \((Y)\), the interest rate \((r)\), and perceived wealth \(\frac{W}{P}\). By the assumption that no distribution effects follow arbitrary changes in individual \(W\)'s, the influence of bond holdings and bond debts must cancel out and the value of \(a\) must be the same for all individuals.

Hence, the market-clearing equations, any two of which are independent, reduce to:

\[
E(Y_o, r, \frac{M_o}{(P)^a}) - Y_o = 0,
\]

\[
B(Y_o, r, \frac{M_o}{(P)^a}) = 0,
\]

\[
L(Y_o, r, \frac{M_o}{(P)^a}) - \frac{M_o}{(P)^a} = 0.
\]

In these equations, \(Y_o\) and \(M_o\) are the exogenously-determined values of commodity output and the money stock. The conditions on the signs of the partial derivatives of the above equations are:

\[
1 > E_1 > 0, \quad E_2 < 0, \quad E_3 > 0,
\]

\[
B_1 > 0, \quad B_2 > 0, \quad B_3 > 0,
\]

\[
L_1 > 0, \quad L_2 < 0, \quad 1 > L_3 > 0.
\]

Following standard comparative-statics procedure, let us eliminate one market equation by Walras' Law and assume an equilibrium at \(P_o\) and \(r_o\). Next,
let us disturb this equilibrium by doubling the money stock. As the last step, we want to assess the system at the Quantity-Theorem solution \((2P_o, r_o)\). Since this experiment does not change the value of \(Y\) or of \(r\), whether or not we have found a new equilibrium depends on whether or not \(\frac{M_o}{(P_o)^{\alpha}}\) is equal to \(\frac{2M_o}{(2P_o)^{\alpha}} = 2 \cdot \frac{M_o}{P_o} \cdot 2^\alpha\). Clearly, this equality only holds if \(\alpha = 1\). If \(0 < \alpha < 1\), the commodity and bond markets remain in a state of excess demand and the money market in a state of excess supply.

To wipe out these market disequilibria while maintaining interest rate \(r_o\) (that is, for money to be neutral), it would be necessary to have \(P\) rise even further. It would have to rise until the new level was exactly \(2^{1/\alpha}\) times the old one. Such a price level would restore the initial level of perceived balances, so that all arguments would have their original values. Notice that if \(\alpha = 0\), the case of extreme money illusion, no such price-level solution can exist. Under extreme money illusion, the price level -- as Patinkin has emphasized -- is indeterminate.

**Geometric Analysis and Dynamic Pressures**

To clarify our analysis and to show the existence of dynamic pressures that push the system toward equilibrium at \((r_o, 2^{1/\alpha} P_o)\), we resort to a diagram. For the case \(\alpha = \frac{1}{2}\), Figure 1 depicts the original and post-disturbance equilibria in terms of the familiar market-equilibrium curves CC, BB, and LL. As our preceding analysis indicates, a doubling of the money stock shifts the price-level co-ordinate of each point of every curve to a location \(2^{1/\alpha}\) (here four)
times its original value.

As a consequence, both the initial equilibrium at \((r_o, P_o)\) and the providional equilibrium at \((r_o, 2P_o)\) lie above the post-disturbance BB and LL curves and below the post-disturbance CC curve. According to the rules of conventional tâtonnment, the conditions of excess demand in each and every market dictate a further increase in \(P\). This is illustrated in Table 1.

![Figure 1](image)

**FIGURE 1**

**TABLE 1**

<table>
<thead>
<tr>
<th>Geometric Location</th>
<th>Market Condition</th>
<th>Movements Which Would Help Clear the Market in Question</th>
</tr>
</thead>
</table>
| Below CC           | Excess demand for commod- | \begin{tabular}{c|c|c} | Change in P | Change in r \\
| ties                  |                           | \end{tabular} |
| Above BB           | Excess demand for bonds   | Up           | Up          |
| Above LL           | Excess supply of money    | Up           | Down        |
Summary

By parametrizing the concept of the degree of money illusion, we have established that in the Patinkin model it is not the complete absence of money illusion \( \alpha = 1 \), but only the absence of extreme money illusion \( \alpha = 0 \) which is necessary for prices to be determinate and money neutral. We have also shown that, whenever there is money illusion of the less-than-extreme type, commodity prices will tend to rise or fall more than in proportion to exogenous changes in the money stock. Moreover, the degree of nonproportionality will vary directly with the extent of the money illusion that exists.